Test answers - Proof

Year: Year 12 · Duration: 60 minutes · /20

Answer to Question 1 (4 marks)

a. Odd numbers have the form $2k+1$; consecutive odd numbers differ by 2. $2n-1$ and $2n+1$ (1)
b. Expand: $(2n-1)^2 = 4n^2 - 4n + 1$ and $(2n+1)^2 = 4n^2 + 4n + 1$; the $\pm 4n$ terms cancel. $(2n-1)^2 + (2n+1)^2 = 8n^2 + 2$ (2). This is $8n^2 + 2$, which is 2 more than the multiple of 8 given by $8n^2$ (1). Since n^2 is an integer, the sum is always 2 more than a multiple of 8.

Using n and $n+2$ is wrong because they need not be odd.

Answer to Question 2 (4 marks)

a. One counter-example is enough to disprove a general statement; try small primes. $n = 7$: $7 + 2 = 9$ is not prime (2) So the statement is false.
b. Choose simple non-zero values of a and b. $a = 1$, $b = 2$: $(1+2)^2 = 9$ but $1^2 + 2^2 = 5$ (2) So $(a+b)^2 \neq a^2 + b^2$ in general; the correct expansion is $a^2 + 2ab + b^2$.

The marks require the calculation as well as the example. In (a), $n = 2$ also works, since $2 + 2 = 4$ is not prime.

Answer to Question 3 (4 marks)

Every integer is either even or odd, so checking these two cases covers all integers. Write each case algebraically and factorise out 2.

If $n = 2k$: $n^2 + n = 4k^2 + 2k = 2(2k^2 + k)$, even (2). If $n = 2k+1$: $(2k+1)^2 + (2k+1) = 4k^2 + 6k + 2 = 2(2k^2 + 3k + 1)$, even (2). Both cases are even so the statement is true.

Alternatively, $n^2 + n = n(n+1)$ is a product of consecutive integers, one of which must be even.

Checking a few values such as 2 and 3 is not a proof.

Answer to Question 4 (4 marks)

In a proof by contradiction, assume the opposite of the statement and show that it leads to something impossible.

Assume there is a largest even number N (1). Then $N+2$ is also even (1) and $N + 2 > N$ (1), which contradicts the assumption. So there is no largest even number

(1).

The same argument shows that there is no largest integer of any kind.

Answer to Question 5 (4 marks)

a. The numbers of seats form an arithmetic sequence with first term 10 and common difference 2. $10 + 2(n-1) = 2n + 8$ (1) This is the required expression.
b. Factorise out 2. $2n + 8 = 2(n + 4)$ (1) Since $n + 4$ is an integer, every row has an even number of seats.
c. Use $S_n = \frac{n}{2}[2a + (n-1)d]$ with $n = 15$, $a = 10$, $d = 2$. Arithmetic series: $\frac{15}{2}(2 \times 10 + 14 \times 2) = \frac{15}{2} \times 48 = 360$ (1 for the formula, 1 for 360) The hall has 360 seats in total.