Worksheet - Probability and the binomial distribution

Year: Year 12 · 20

Show your working.

Question 1

Question 1 – Probability rules *(Section A · Skills practice)*

A and B are events with $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$.

a. Find $P(A \cup B)$. [2 marks]
b. Find $P(A' \cap B')$. [2 marks]
c. Find $P(A|B)$. [2 marks]
d. Are A and B independent? Justify. [2 marks]

Question 2

Question 2 – A Venn diagram of sports *(Section A · Skills practice)*

Of 40 students, 22 play football (F), 18 play tennis (T) and 9 play both.

a. Draw a Venn diagram and find how many play neither sport. [3 marks]
b. Find the probability that a randomly chosen student plays football only. [2 marks]
c. Find $P(T|F)$. [2 marks]
d. Find the probability that a student plays exactly one sport. [2 marks]

Question 3

Question 3 – Tree diagram without replacement *(Section A · Skills practice)*

A bag contains 5 red and 3 blue counters. Two counters are taken at random without replacement.

a. Draw a tree diagram showing the probabilities. [3 marks]
b. Find the probability that both are red. [2 marks]
c. Find the probability that the counters are different colours. [3 marks]
d. Find the probability that at least one is blue. [2 marks]

Question 4

Question 4 – Mutually exclusive and independent events *(Section A · Skills practice)*

$P(X) = 0.3$ and $P(Y) = 0.5$.

a. Find $P(X \cup Y)$ if X and Y are mutually exclusive. [2 marks]
b. Find $P(X \cup Y)$ if X and Y are independent. [3 marks]
c. Explain why two events with non-zero probabilities cannot be both mutually exclusive and independent. [2 marks]

Question 5

Question 5 – The binomial distribution *(Section A · Skills practice)*

$X \sim B(8, 0.3)$.

a. Find $P(X = 2)$. [2 marks]
b. Find $P(X \leq 1)$. [3 marks]
c. Find $P(X \geq 3)$. [3 marks]
d. Find $E(X)$ and $\mathrm{Var}(X)$. [2 marks]

Question 6

Question 6 – Is the binomial model suitable? *(Section A · Skills practice)*

For each situation decide whether X can be modelled by a binomial distribution. Give the parameters or explain why not.

a. X is the number of sixes in 10 rolls of a fair die. [2 marks]
b. X is the number of red cards in a hand of 5 cards dealt from a pack without replacement. [2 marks]
c. X is the number of attempts until the first six. [2 marks]
d. State the four conditions for a binomial model. [2 marks]

Question 7

Question 7 – The weather week *(Section B · Real-life practice)*

On any day in a particular town the probability of rain is 0.3, independently of other days.

a. Find the probability of no rain for three days in a row. [2 marks]
b. Find the probability of exactly 2 rainy days in 5 days. [3 marks]
c. Find the probability of at least one rainy day in a week. [3 marks]
d. Find the expected number of dry days in a 30-day month. [2 marks]

Question 8

Question 8 – Quality control *(Section B · Real-life practice)*

A factory makes light bulbs. 4% of bulbs are faulty, independently. A box contains 20 bulbs.

a. Define a suitable binomial model. [2 marks]

b. Find the probability that a box has no faulty bulbs. [2 marks]
c. Find the probability that a box has at most 2 faulty bulbs. [4 marks]
d. The factory refunds if more than 2 bulbs are faulty. Find the probability of a refund and the expected number of faulty bulbs per box. [3 marks]

Question 9

Question 9 – A medical test *(Section B · Real-life practice)*

2% of a population have a disease. A test gives a positive result for 95% of people with the disease and for 3% of people without it.

a. Draw a tree diagram. [3 marks]
b. Find the probability that a randomly chosen person tests positive. [3 marks]
c. Given that a person tests positive, find the probability that they have the disease. [3 marks]
d. Explain why the answer to part (c) is surprisingly low. [2 marks]

Question 10

Question 10 – Guessing in a test *(Section B · Real-life practice)*

A multiple-choice test has 12 questions, each with 4 options of which one is correct. Zoe guesses every answer.

a. State the distribution of the number of correct answers. [1 mark]
b. Find the probability that she gets exactly 3 correct. [2 marks]
c. The pass mark is 6. Find the probability that she passes by guessing. [4 marks]
d. Find her expected score and comment. [2 marks]

Question 11

Question 11 – Late buses *(Section B · Real-life practice)*

The probability that Maya\'s bus is late on any day is 0.2, independently of other days. She takes the bus on 5 school days.

a. Find the probability that the bus is never late. [2 marks]
b. Find the probability that it is late exactly twice. [2 marks]
c. Find the probability that it is late on at least 3 days. [4 marks]
d. The school sends a letter if the bus is late on 3 or more days in a week. About how many letters would be sent in a 38-week school year? [2 marks]

Question 12

Question 12 – Free throws *(Section B · Real-life practice)*

A basketball player scores each free throw with probability 0.7, independently. She

takes 10 shots.

a. Find the probability she scores exactly 8. [2 marks]
b. Find the probability she scores 8 or more. [3 marks]
c. Find the probability she scores 6 or fewer. [2 marks]
d. Find the mean and standard deviation of the number of baskets. [2 marks]

Question 13

Question 13 – Two dice *(Section B · Real-life practice)*

Two fair dice are rolled and the scores are added.

a. Find the probability that the total is 7. [2 marks]
b. Find the probability that at least one die shows a 6. [2 marks]
c. Given that at least one die shows a 6, find the probability that the total is at least 10. [3 marks]
d. Are the events "at least one six" and "total at least 10" independent? [3 marks]

Question 14

Question 14 – Picking raffle prizes *(Section B · Real-life practice)*

A drawer holds 6 red and 4 green socks. Three socks are taken at random without replacement.

a. Find the probability that all three are red. [3 marks]
b. Find the probability that exactly two are green. [4 marks]
c. Find the probability that at least one is green. [2 marks]
d. Explain why the number of green socks is not binomial. [1 mark]

Question 15

Question 15 – Travel to sixth form *(Section C · Problem solving)*

150 sixth-form students were asked how they travel to school. The results are shown.

	Walk	Bus	Car	Total
Year 12	30	25	15	70
Year 13	20	35	25	80
Total	50	60	40	150

a. Find $P(\text{bus})$ and $P(\text{Year 13 and car})$. [2 marks]
b. Find $P(\text{car} \mid \text{Year 13})$. [2 marks]
c. Find $P(\text{Year 12} \mid \text{walk})$. [2 marks]
d. Test whether "Year 13" and "car" are independent. [4 marks]

Question 16

Question 16 – How many trials? *(Section C · Problem solving)*

Each time an app is opened there is a probability of 0.1 that a new advert is shown. Opens are independent.

Number of opens n	10	20	30	40	44
$P(\text{at least one advert})$	0.6513	0.8784	0.9576	0.9852	0.9903

a. Show that $P(\text{at least one}) = 1 - 0.9^n$ and check the entry for $n = 20$. [3 marks]
b. Find the smallest n such that this probability exceeds 0.99. [4 marks]
c. Verify with $n = 43$ and $n = 44$. [2 marks]
d. Find the expected number of adverts in 44 opens. [1 mark]

Question 17

Question 17 – Which machine made the faulty item? *(Section C · Problem solving)*

Three machines A, B and C make 50%, 30% and 20% of a factory\'s output. Their defect rates are 1%, 2% and 4%.

Machine	Share	Defect rate	Share × rate
A	0.5	0.01	0.005
B	0.3	0.02	0.006
C	0.2	0.04	0.008

a. Find the probability that an item chosen at random is defective. [3 marks]
b. Given that an item is defective, find the probability it came from machine C. [3 marks]
c. Find the probability that two items chosen at random are both defective. [2 marks]
d. How many defectives are expected in a batch of 5 000 items? [1 mark]

Question 18

Question 18 – Using binomial tables *(Section C · Problem solving)*

$X \sim B(15, 0.4)$. Some cumulative probabilities $P(X \le r)$ are given.

r	3	4	5	6	7	8
$P(X \le r)$	0.0905	0.2173	0.4032	0.6098	0.7869	0.9050

a. Use the table to find $P(X \le 4)$ and $P(X = 6)$. [3 marks]
b. Find $P(X \ge 8)$. [2 marks]
c. Find the smallest value c such that $P(X \le c) > 0.5$. [2 marks]
d. Find the mean and standard deviation, and comment on how the mean compares

with the value from part (c). [3 marks]

Question 19

Question 19 – A three-set Venn diagram *(Section C · Problem solving)*

In a club of 60 members, 34 swim (S), 28 cycle (C) and 20 run (R). 12 swim and cycle, 9 swim and run, 8 cycle and run, and 5 do all three.

a. Find the number who do none of the sports. [4 marks]
b. Find the number who do exactly one sport. [4 marks]
c. Find the probability that a randomly chosen member cycles given they run. [2 marks]
d. Find the probability that a member does exactly two sports. [2 marks]

Question 20

Question 20 – Challenge: finding n and p *(Section D · Challenge)*

$X \sim B(n, p)$ has mean 6 and variance 4.2.

a. Find n and p. [4 marks]
b. Find $P(X = 6)$. [2 marks]
c. Show that $\dfrac{P(X = r+1)}{P(X = r)} = \dfrac{(n-r)p}{(r+1)q}$ and use it to show that 6 is the most likely value. [5 marks]