# Lesson - Kinematics

Year: Year 12

Kinematics describes motion without asking what causes it. In Year 12 you study straight-line motion with constant acceleration, motion graphs, and then use calculus when the acceleration varies. Mechanics always starts with units and a clear sign convention.

**Starter activity**

You throw a ball straight up and catch it again. At what moment is its speed zero? What is its acceleration at that moment: zero, upwards or downwards? Discuss with a partner and then check by sketching a velocity-time graph for the whole flight, from release to catch. What shape does the graph have, and what does its gradient tell you? Where would you read the maximum height from the graph?

## Quantities and units

**Definition: scalar and vector**

**Scalars** have size only (distance, speed, time). **Vectors** have size and direction (displacement, velocity, acceleration). In one dimension choose a positive direction and use signs: a negative velocity means moving in the negative direction.

**Key fact: SI units**

The base units are the metre (m), kilogram (kg) and second (s). Velocity is m $\mathrm{s}^{-1}$, acceleration m $\mathrm{s}^{-2}$, force the newton: $1\ \mathrm{N} = 1\ \mathrm{kg\ m\ s^{-2}}$. To convert km/h to m/s multiply by $\dfrac{1000}{3600} = \dfrac{5}{18}$: $72$ km/h $= 20$ m/s.

## Motion graphs

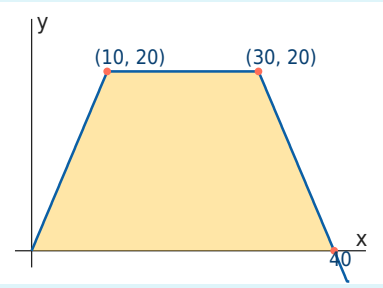
**Rule: reading motion graphs**

- Displacement-time graph: the **gradient** is the velocity.
- Velocity-time graph: the **gradient** is the acceleration and the **area** under the graph is the displacement (the total distance if the velocity stays positive).

**Worked example 1**

**A car accelerates from rest to $20$ m/s in 10 s, travels at $20$ m/s for 20 s, then decelerates to rest in 10 s. Find the distance and average speed.**

Area of the trapezium $= \frac{1}{2}(20 + 40) \times 20 = 600$ m. Average speed $= \dfrac{600}{40} = 15$ m/s.




## Constant acceleration (suvat)

### Rule: the suvat equations

With initial velocity $u$, final velocity $v$, acceleration $a$, displacement $s$ and time $t$:

$$v = u + at, \quad s = \frac{(u+v)}{2}t, \quad s = ut + \tfrac{1}{2}at^2, \quad s = vt - \tfrac{1}{2}at^2, \quad v^2 = u^2 + 2as.$$

Only use them when the acceleration is **constant**.

### Note: where they come from

The velocity-time graph for constant acceleration is a straight line from $(0, u)$ to $(t, v)$, and the area is a trapezium: $s = \frac{1}{2}(u + v)t$. Substituting $v = u + at$ gives $s = ut + \frac{1}{2}at^2$.

### Method: choosing the suvat equation

1. List $s, u, v, a, t$ and mark what is known.
2. Mark what is wanted.
3. Choose the equation that contains exactly those four quantities.

### Worked example 2

**A car brakes from $12$ m/s to rest over $36$ m. Find $a$ and the time taken.**

$v^2 = u^2 + 2as$: $0 = 144 + 72a$, so $a = -2$ m/s$^2$. Then $v = u + at$ gives $0 = 12 - 2t$, $t = 6$ s.

## Vertical motion under gravity

### Rule: free motion under gravity

Ignoring air resistance, an object moving freely has acceleration $g = 9.8$ m s$^{-2}$

downwards. Take upward as positive, so $a = -9.8$. At the highest point $v = 0$.

### Worked example 3

**A ball is thrown up at $14.7$ m/s. Find the time to the top, the greatest height and the total flight time.**

$0 = 14.7 - 9.8t$, so $t = 1.5$ s. Height $= \dfrac{14.7^2}{2 \times 9.8} = 11.0$ m. By symmetry the total flight time is 3 s.

## Variable acceleration and calculus

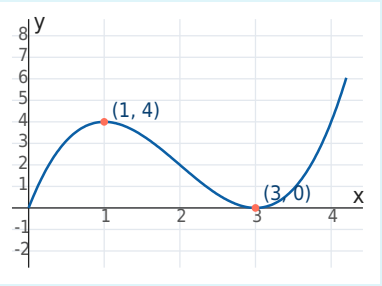
### Rule: calculus in kinematics

If the displacement is $s(t)$, then $v = \dfrac{ds}{dt}$ and $a = \dfrac{dv}{dt} = \dfrac{d^2s}{dt^2}$. Going backwards, $v = \int a\,dt$ and $s = \int v\,dt$, with the constant found from initial conditions. The particle is **at rest** when $v = 0$; the maximum velocity occurs when $a = 0$.

### Worked example 4

**$s = t^3 - 6t^2 + 9t$. Find when the particle is at rest, and the acceleration.**

$v = 3t^2 - 12t + 9 = 3(t-1)(t-3)$, so it is at rest at $t = 1$ and $t = 3$, where $s = 4$ and $s = 0$. The acceleration is $a = 6t - 12$.




### Method: distance travelled

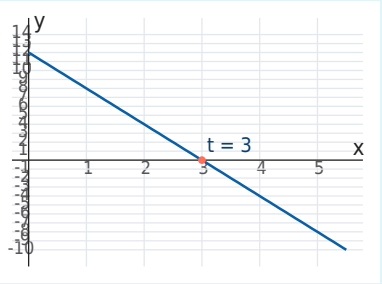
1. Find the times when $v = 0$.
2. Split the journey at each turning point.
3. Find the displacement of each part and add the sizes.

### Worked example 5

**$v = 12 - 4t$. Find the distance travelled in the first 5 seconds.**

The particle stops at $t = 3$ (displacement $\int_0^3 (12 - 4t)\,dt = 18$ m), then moves back. At

$t = 5$, $s = \left[12t - 2t^2\right]_0^5 = 10$, so it returned 8 m and the total distance is $18 + 8 = 26$ m.




**Watch out**

Do not use suvat when the acceleration is not constant. A deceleration is a negative $a$ in your chosen direction. Do not mix km/h and m/s. After a change of direction, distance and displacement differ: read the question.

**Worked example 6: Using v-t graph areas for an unknown time**

**A car accelerates from rest to 15 m/s in 6 s, travels at 15 m/s, then brakes to rest in 5 s. The total distance is 600 m. Find the time at constant speed.**

Stage 1 distance $= \frac{1}{2} \times 6 \times 15 = 45$ m. Stage 3 distance $= \frac{1}{2} \times 5 \times 15 = 37.5$ m. The remaining distance at constant speed is $600 - 45 - 37.5 = 517.5$ m, taking $\dfrac{517.5}{15} = 34.5$ s. The total journey time is $6 + 34.5 + 5 = 45.5$ s and the average speed is $\dfrac{600}{45.5} = 13.2$ m/s. Whenever a graph is made of straight lines, split the area into triangles and rectangles.

**Worked example 7: Choosing the right equation**

**A cyclist slows from 10 m/s to 4 m/s in 8 s. Find the deceleration and the distance covered, and then the distance covered in the first 3 s.**

$a = \dfrac{4 - 10}{8} = -0.75$ m/s$^2$. Distance $= \frac{1}{2}(10 + 4) \times 8 = 56$ m. In the first 3 s: $s = 10 \times 3 - \frac{1}{2} \times 0.75 \times 9 = 30 - 3.375 = 26.625$ m. The choice of equation depends on which four of the five quantities $s, u, v, a, t$ are known or wanted: list them and pick the equation that contains all four and not the fifth.

**Worked example 8: Time of flight from a height**

**A ball is dropped from a window 6 m above the ground. Find the time to fall and the impact speed.**

$6 = \frac{1}{2} \times 9.8t^2$, so $t^2 = 1.2245$ and $t = 1.11$ s. Impact speed $v = 9.8 \times 1.107 = 10.8$ m/s. Alternatively $v^2 = 2 \times 9.8 \times 6 = 117.6$, $v = 10.84$ m/s ✓. Using $v^2 = u^2 + 2as$ avoids

the time, so it is quicker when only the speed is needed. Remember to state the direction (downwards) when giving a velocity rather than a speed.

### Worked example 9: Integration with initial conditions

**The acceleration of a particle is $a = 4 - 2t$ m/s$^2$. At $t = 0$, $v = 0$ and $s = 0$. Find when it returns to its starting point.**

$v = 4t - t^2$ and $s = 2t^2 - \dfrac{t^3}{3}$. It returns to the start when $s = 0$: $t^2\left(2 - \dfrac{t}{3}\right) = 0$, so $t = 6$ s. Check the motion: $v > 0$ for $0$ (moving forwards), $v = 0$ at $t = 4$ with $s = 32 - \dfrac{64}{3} = 10.67$ m, and $v$ for $t > 4$ so the particle comes back. The furthest distance forward is 10.67 m.

### Worked example 10: Comparing two objects with the same acceleration

**Two stones are dropped from the same height one second apart. How far apart are they after the first has fallen for 3 s?**

The first stone has fallen $\frac{1}{2} \times 9.8 \times 3^2 = 44.1$ m. The second has fallen for 2 s: $\frac{1}{2} \times 9.8 \times 4 = 19.6$ m. The separation is $44.1 - 19.6 = 24.5$ m. The separation keeps growing because the first stone is faster: after 3 s its speed is 29.4 m/s and the second is 19.6 m/s, a difference of 9.8 m/s that stays constant.

### Worked example 11: Speed, velocity and a sign change

**A particle moves with velocity $v = 6 - 2t$ m/s from $t = 0$ where $s = 0$. Find the velocity, speed and displacement at $t = 5$, and the total distance travelled in the first 5 seconds.**

At $t = 5$: $v = 6 - 10 = -4$ m/s, so the speed is 4 m/s and the particle is moving in the negative direction. Displacement $s = 6t - t^2 = 30 - 25 = 5$ m. The velocity is zero at $t = 3$ when $s = 18 - 9 = 9$ m, so the particle first goes 9 m forward and then comes back 4 m to $s = 5$. The total distance is $9 + 4 = 13$ m, which is not the same as the displacement of 5 m. A particle that reverses direction always travels further than its displacement suggests.

### Worked example 12: Constant acceleration in a context with units

**A motorcyclist increases speed from 36 km/h to 90 km/h in 10 s. Find the acceleration in m/s$^2$ and the distance travelled.**

Convert: $36\ \text{km/h} = 10$ m/s and $90\ \text{km/h} = 25$ m/s (divide by 3.6). Acceleration $= \dfrac{25 - 10}{10} = 1.5$ m/s$^2$. Distance $= \frac{1}{2}(10 + 25) \times 10 = 175$ m. Always convert to m/s before using the equations, since the standard value $g = 9.8$ m/s$^2$ and the constant-acceleration formulae assume metres and seconds.

## Method: solving a kinematics problem

1. Draw a diagram and choose a positive direction.
2. List $s$, $u$, $v$, $a$ and $t$ with their signs and mark the unknown.
3. Pick the equation that contains four of the five variables (or use a graph, or calculus if acceleration varies).
4. Solve, check the sign and size of the answer and state the units.

## Watch out: common mistakes in kinematics

- Using the suvat equations when the acceleration is not constant. Use calculus instead.
- Mixing speed (always positive) with velocity (has a sign) and distance with displacement.
- Using the wrong sign for $g$: if upwards is positive, $a = -9.8$.
- Forgetting units: convert km/h to m/s by dividing by 3.6 before using $g = 9.8$ m/s$^2$.
- Taking the area under a velocity-time graph below the axis as positive distance without adding the absolute value.

## Note: choosing between graphs and equations

A velocity-time graph gives three ideas at once: the gradient is the acceleration, the area is the displacement, and the shape shows when the motion changes. For straight-line sections you can use areas of triangles, rectangles and trapezia; for curved graphs integrate. When the acceleration is constant, the five suvat equations are quicker. When the acceleration depends on $t$, differentiate to go from displacement to velocity to acceleration and integrate (with a boundary condition for the constant) to go the other way. Choosing the method that fits the information is the key skill in this chapter.

## Did you know?

Galileo Galilei is said to have dropped balls from the Leaning Tower of Pisa around 1590 to show that objects of different mass fall at the same rate, contradicting Aristotle, though he probably used ramps in his careful experiments. The constant acceleration due to gravity near the Earth\'s surface, $g \approx 9.8$ m/s$^2$, means that an object falling for 1 second is moving at 9.8 m/s and falls only 4.9 m, but after 5 seconds it moves at 49 m/s and has fallen 122.5 m.

## Otto's tip

Write "SUVAT" down the page, fill in what you know, and the equation chooses itself. State your positive direction ("take upwards as positive").