Answers - Kinematics

Year: Year 12

Answer to Question 1

a. Acceleration is the gradient of the velocity-time graph: change in velocity divided by time. Stage 1: $a = \dfrac{12 - 0}{6} =$ **2 m/s^2**. Stage 3: $a = \dfrac{0 - 12}{4} = -3$, a deceleration of **3 m/s^2**.

b. Distance is the area under the velocity-time graph. Stage 1: area of triangle $\frac{1}{2} \times 6 \times 12 =$ **36 m**. Stage 2: $12 \times 20 =$ **240 m**. Stage 3: $\frac{1}{2} \times 4 \times 12 =$ **24 m**.

c. Total distance $= 36 + 240 + 24 =$ **300 m** in $6 + 20 + 4 = 30$ s. Average speed $= \dfrac{300}{30} =$ **10 m/s**. The average speed is less than the top speed because the lift spends time speeding up and slowing down.

Answer to Question 2

a. Here u, a and t are known, so use $v = u + at$ and $s = ut + \frac{1}{2}at^2$. $v = u + at = 3 + 10 =$ **13 m/s**. $s = ut + \frac{1}{2}at^2 = 15 + 25 =$ **40 m**. After 5 s the particle moves at 13 m/s and has travelled 40 m.

b. u, v and s are known; $s = \frac{1}{2}(u + v)t$ gives t directly. $s = \frac{1}{2}(u + v)t$: $66 = 11t$, so $t =$ **6 s**. $a = \dfrac{v - u}{t} = \dfrac{12}{6} =$ **2 m/s^2**. Check with $v^2 = u^2 + 2as$: $25 + 2 \times 2 \times 66 = 289 = 17^2$ ✓.

c. s, t and a are known and u is wanted, so use $s = ut + \frac{1}{2}at^2$. $s = ut + \frac{1}{2}at^2$: $48 = 4u + \frac{1}{2} \times 1.5 \times 16 = 4u + 12$, so $u =$ **9 m/s**. Check: $9 \times 4 + 0.75 \times 16 = 48$ ✓.

Answer to Question 3

a. List the quantities: $u = 20$, $v = 0$, $a = -4$; the time is not needed, so use $v^2 = u^2 + 2as$. $v^2 = u^2 + 2as$: $0 = 400 - 8s$, so $s =$ **50 m**. The braking distance is 50 m.

b. Use $v = u + at$ with $v = 0$. $v = u + at$: $0 = 20 - 4t$, $t =$ **5 s**. The car stops after 5 s.

c. Use $v^2 = u^2 + 2as$ with $s = 30$. $v^2 = 400 - 2 \times 4 \times 30 = 160$, so $v = \sqrt{160} =$ **12.6 m/s**. After 30 m of braking the car is still moving at about 12.6 m/s.

Answer to Question 4

a. Take upwards as positive, so $a = -9.8$; at the highest point $v = 0$. Use $v^2 = u^2 + 2as$. At the top $v = 0$: $0 = 196 - 19.6s$, so $s =$ **10 m**. The ball rises 10 m.

b. Use $v = u + at$ with $v = 0$; by symmetry the time down equals the time up. $0 = 14 - 9.8t$ gives $t = 1.43$ s up. Total flight $= 2 \times 1.43 =$ **2.86 s**. The ball is in the air for about 2.86 s.

c. Substitute $t = 2$ into $v = u + at$ and $s = ut + \frac{1}{2}at^2$. $v = 14 - 9.8 \times 2 = -5.6$ **m/s** (5.6 m/s downwards). $s = 14 \times 2 - 4.9 \times 4 = 28 - 19.6 =$ **8.4 m**. The negative velocity shows the ball is already falling, at a height of 8.4 m.

Answer to Question 5

a. Velocity is the derivative of displacement and acceleration is the derivative of velocity. $v = \dfrac{ds}{dt} = 6t^2 - 18t + 12$ and $a = \dfrac{dv}{dt} = 12t - 18$.

b. The particle is at rest when $v = 0$; factorise the velocity. $6(t^2 - 3t + 2) = 6(t - 1)(t - 2) = 0$ at $t = 1$ and $t = 2$. So the particle is at rest at $t = 1$ and $t = 2$.

c. Substitute the times into s and a. $s(1) = 2 - 9 + 12 =$ **5 m**, $s(2) = 16 - 36 + 24 =$ **4 m**. $a(1) = 12 - 18 = -6$ **m/s**^{2}. At $t = 1$ the particle is 5 m from the origin and its velocity is decreasing.

Answer to Question 6

a. Velocity is the integral of acceleration; the initial velocity gives the constant. $v = \int (6t - 4)dt = 3t^2 - 4t + c$. $v(0) = 3$ so $c = 3$: $v = 3t^2 - 4t + 3$. This is the velocity at time t.

b. Displacement is the integral of velocity; the initial displacement gives the constant. $s = \int v\,dt = t^3 - 2t^2 + 3t + d$. $s(0) = 0$ so $d = 0$: $s = t^3 - 2t^2 + 3t$. This is the displacement at time t.

c. Substitute $t = 2$ into both expressions. $v(2) = 12 - 8 + 3 =$ **7 m/s**. $s(2) = 8 - 8 + 6 =$ **6 m**. At $t = 2$ the particle moves at 7 m/s and is 6 m from its starting point.

Answer to Question 7

a. Use $t = \dfrac{v - u}{a}$ for the acceleration and deceleration stages; 4 minutes is 240 s. Stage 1: $t = \dfrac{25}{0.5} = 50$ s. Stage 2: $4 \times 60 = 240$ s. Stage 3: $t = \dfrac{25}{1} = 25$ s. Total 315 s.

b. The distance is the area under the velocity-time graph, made of two triangles and a rectangle. Stage 1: $\frac{1}{2} \times 25 \times 50 = 625$ m. Stage 2: $25 \times 240 = 6000$ m. Stage 3: $\frac{1}{2} \times 25 \times 25 = 312.5$ m. Total **6 937.5 m**. The stations are about 6.94 km apart.
c. Average speed is total distance divided by total time; multiply by 3.6 to convert m/s to km/h. Average speed $= \dfrac{6937.5}{315} =$ **22.0 m/s** $= 22.02 \times 3.6 =$ **79.3 km/h**.
d. A trapezium: rising line from $(0, 0)$ to $(50, 25)$, horizontal to $(290, 25)$, falling to $(315, 0)$. The area of this trapezium is the distance found in part (b).

Answer to Question 8

a. During the reaction time the car moves at constant speed, so distance = speed × time. $15 \times 0.7 =$ **10.5 m**. The thinking distance is 10.5 m.
b. Use $v^2 = u^2 + 2as$ with $u = 15$, $v = 0$, $a = -5$. $0 = 225 - 10s$, so $s =$ **22.5 m**. The braking distance is 22.5 m.
c. Compare the total stopping distance with the distance to the hazard. Total stopping distance $= 10.5 + 22.5 = 33$ m > 30 m, so she hits the hazard.
d. Only the distance left after the thinking distance is available for braking; use $v^2 = u^2 + 2as$. Braking distance available $= 30 - 10.5 = 19.5$ m.
$v^2 = 225 - 2 \times 5 \times 19.5 = 30$, so $v =$ **5.48 m/s**. She hits the hazard at about 5.48 m/s, much slower than her original speed.

Answer to Question 9

a. Take downwards as positive with $u = 0$ and $a = 9.8$; use $s = ut + \frac{1}{2}at^2$, then $v = u + at$. $45 = \frac{1}{2} \times 9.8t^2$, $t^2 = 9.18$, $t =$ **3.03 s**. $v = 9.8 \times 3.03 =$ **29.7 m/s**. The stone takes about 3.03 s and hits the water at about 29.7 m/s.
b. Now $u = 5$ (downwards) with $a = 9.8$ and $s = 45$. $s = ut + \frac{1}{2}at^2$: $45 = 5t + 4.9t^2$, so $4.9t^2 + 5t - 45 = 0$. This is the required equation.
c. Use the quadratic formula and take the positive root, then use $v = u + at$.
$t = \dfrac{-5 + \sqrt{25 + 882}}{9.8} = \dfrac{-5 + 30.12}{9.8} =$ **2.56 s**. $v = 5 + 9.8 \times 2.563 =$ **30.1 m/s**.
The second stone arrives about 0.47 s sooner and slightly faster.

Answer to Question 10

a. Use $v = u + at$ with $u = 0$. $v = 1.2 \times 3 =$ **3.6 m/s**. The maximum speed is 3.6 m/s.
b. The lift slows from 3.6 m/s to 0 in 4 s. $a = \dfrac{0 - 3.6}{4} = -0.9$: deceleration **0.9** ^{2} **m/s**.
c. The height climbed is the area under the velocity-time graph (two triangles and a

rectangle). Distances: $\frac{1}{2} \times 3 \times 3.6 = 5.4$ m; $3.6 \times 10 = 36$ m; $\frac{1}{2} \times 4 \times 3.6 = 7.2$ m. Total **48.6 m**. The lift climbs 48.6 m.

d. Divide the total distance by the total time. Time $= 3 + 10 + 4 = 17$ s. Average speed $= \dfrac{48.6}{17} =$ **2.86 m/s**. The average speed is about 2.86 m/s.

Answer to Question 11

a. A starts 12 m ahead of B's starting point and moves at a constant 7 m/s. $12 + 7t$. This is A's distance from B's starting point.

b. B starts from rest, so use $s = \frac{1}{2}at^2$. $s = \frac{1}{2} \times 1.5t^2 = 0.75t^2$. This is B's distance from her starting point.

c. B catches A when both distances are equal; solve with the formula and keep the positive root. $0.75t^2 = 12 + 7t$, so $0.75t^2 - 7t - 12 = 0$.
$t = \dfrac{7 + \sqrt{49 + 36}}{1.5} = \dfrac{7 + 9.22}{1.5} =$ **10.8 s**. B catches A after about 10.8 s.

d. Substitute $t = 10.81$ into $s = 0.75t^2$ and $v = at$. $s = 0.75 \times 10.81^2 =$ **87.7 m**. $v = 1.5 \times 10.81 =$ **16.2 m/s**, faster than A. B has run about 87.7 m when she catches A.

Answer to Question 12

a. Integrate the acceleration and use the initial speed to find the constant. $v = \int (0.6 - 0.04t)dt = 0.6t - 0.02t^2 + c$, $v(0) = 2$. This is the required expression.

b. Substitute $t = 15$ into v and into a. $v(15) = 2 + 9 - 4.5 =$ **6.5 m/s**. $a(15) = 0.6 - 0.6 = 0$: the lorry has reached its maximum speed.

c. The distance is the integral of the velocity from 0 to 15 (the velocity is always positive here).
$s = \int_0^{15} (2 + 0.6t - 0.02t^2)dt = \left[2t + 0.3t^2 - \dfrac{0.02}{3}t^3\right]_0^{15} = 30 + 67.5 - 22.5 =$ **75 m**. The lorry travels 75 m.

d. Divide the total distance by the total time. $\dfrac{75}{15} =$ **5 m/s**. The average speed is 5 m/s.

Answer to Question 13

a. At the highest point $v = 0$; use $v^2 = u^2 + 2as$ and $v = u + at$ with $a = -9.8$. $0 = 1600 - 19.6s$, $s =$ **81.6 m**. $t = \dfrac{40}{9.8} =$ **4.08 s**. The rocket explodes 81.6 m up, about 4.08 s after launch.

b. Substitute $t = 3$ into $s = ut + \frac{1}{2}at^2$. $s = 40 \times 3 - 4.9 \times 9 = 120 - 44.1 =$ **75.9 m**.

After 3 s the rocket is 75.9 m above the ground.

c. $40t - 4.9t^2 > 70 \Rightarrow 4.9t^2 - 40t + 70$. $t = \dfrac{40 \pm \sqrt{1600 - 1372}}{9.8} = \dfrac{40 \pm 15.1}{9.8}$, giving $t = 2.54$ and 5.62. Time above 70 m $= 5.62 - 2.54 =$ **3.08 s**. (The ascent and descent each pass 70 m: the rocket is above 70 m on both sides of the top.)

Answer to Question 14

a. Use $s = ut + \frac{1}{2}at^2$ with $u = 20$, $a = 0.8$, $s = 300$.
$300 = 20t + \frac{1}{2} \times 0.8t^2 = 20t + 0.4t^2$. Divide by 0.4: $t^2 + 50t - 750 = 0$. This is the required equation.

b. Use the quadratic formula and take the positive root. $t = \dfrac{-50 + \sqrt{2500 + 3000}}{2} =$ **12.08 s**. The car takes about 12.1 s.

c. Use $v = u + at$. $v = 20 + 0.8 \times 12.08 =$ **29.7 m/s**. The final speed is about 29.7 m/s.

d. This equation does not use t, so it is an independent check.
$v^2 = 400 + 2 \times 0.8 \times 300 = 880$, $v = \sqrt{880} = 29.66$ ✓. Both methods give the same final speed.

Answer to Question 15

a. Acceleration is the change in velocity divided by the time taken. Stage 1: $\dfrac{6}{8} = 0.75$ m/s^2. Stage 3: $\dfrac{6 - 2}{10} = 0.4$ m/s^2 deceleration.

b. The distance in each stage is the area under that part of the velocity-time graph. Stage 1: $\frac{1}{2} \times 8 \times 6 = 24$ m. Stage 2: $6 \times 40 = 240$ m. Stage 3: $\frac{6+2}{2} \times 10 = 40$ m. Stage 4: $2 \times 30 = 60$ m.

c. Add the distances and divide by the total time $8 + 40 + 10 + 30 = 88$ s. Total $= 24 + 240 + 40 + 60 = 364$ m in 88 s. Average speed $= \dfrac{364}{88} =$ **4.14 m/s**.

d. 4.14 m/s is a little more than 4 m/s, so she is right to the nearest m/s but slightly underestimates.

Answer to Question 16

a. Integrate v and use $s = 0$ at $t = 0$. $s = \int (t^2 - 8t + 12)dt = \dfrac{t^3}{3} - 4t^2 + 12t$ (since $s(0) = 0$). This is the required expression.

b. Substitute each time into s. $s(2) = \dfrac{8}{3} - 16 + 24 = \dfrac{32}{3} = 10.67$ m.

$s(6) = 72 - 144 + 72 = 0$. $s(8) = \frac{512}{3} - 256 + 96 = \frac{32}{3} = 10.67$ m.

c. The particle changes direction at $t = 2$ and $t = 6$ (where $v = 0$), so add the distances of each stage. Forward 10.67 m, then back 10.67 m, then forward 10.67 m: total **32 m**. The total distance is 32 m, although the final displacement is only 10.67 m.

d. Acceleration is the derivative of velocity. $a = 2t - 8 = 0$ at $t = 4$. The velocity has its minimum (-4 m/s) there, so the acceleration changes sign: negative before and positive after.

Answer to Question 17

a. Use $s = ut + \frac{1}{2}at^2$ for the motion from A to B and from A to C. $60 = 4u + 8a$, i.e. $u + 2a = 15$. $210 = 10u + 50a$, i.e. $u + 5a = 21$.

b. Subtract the equations to eliminate u. Subtract: $3a = 6$, so $a = 2$ and $u = 11$. So the car passes A at 11 m/s with acceleration 2 m/s^2.

c. Use $v = u + at$ with $t = 4$ and $t = 10$. At B: $11 + 8 =$ **19 m/s**. At C: $11 + 20 =$ **31 m/s**.

d. The time from B to C is $10 - 4 = 6$ s. $210 - 60 =$ **150 m**. Time 6 s. Average speed $= 25$ m/s (the mean of 19 and 31). For constant acceleration the average speed is the mean of the initial and final speeds.

Answer to Question 18

a. Use $v^2 = u^2 + 2as$ with $u = 12$, $v = 0$, $a = -9.8$. $0 = 144 - 19.6s$, $s =$ **7.35 m**. The ball rises 7.35 m above the cliff top.

b. The sea is 25 m below the starting point, so the displacement is -25 m. Displacement $-25 = 12t - 4.9t^2$, so $4.9t^2 - 12t - 25 = 0$.

$t = \frac{12 + \sqrt{144 + 490}}{9.8} = \frac{12 + 25.18}{9.8} =$ **3.79 s**.

c. Use $v = u + at$ with the time from part (b). $v = 12 - 9.8 \times 3.794 = -25.2$: speed **25.2 m/s**. The ball hits the sea at about 25.2 m/s.

d. The ball goes up, comes back down to the cliff top and then falls 25 m to the sea. Up 7.35 m, down 7.35 m (back to the cliff), then down 25 m: total $= 14.7 + 25 =$ **39.7 m**. The total distance is 39.7 m, while the displacement is only 25 m downwards.

Answer to Question 19

a. Use $v = u + at$ with $u = V$, $v = 0$, $a = -0.5$. $0 = V - 0.5t$, so $t = 2V$. This is the required time.

b. Find the area under each part of the velocity-time graph. Stage 1: $\frac{1}{2} \times 40 \times V = 20V$. Stage 3: $\frac{1}{2} \times 2V \times V = V^2$. Stage 2 lasts

$300 - 40 - 2V = 260 - 2V$ s, covering $V(260 - 2V) = 260V - 2V^2$. Total $= 20V + 260V - 2V^2 + V^2 = 280V - V^2$. This is the required expression.

c. Set the total distance equal to 8000 m and solve the quadratic.
$280V - V^2 = 8000 \Rightarrow V^2 - 280V + 8000 = 0 \Rightarrow V = \dfrac{280 - \sqrt{46400}}{2} =$ **32.3 m/s** (the other root, 247.7, would make stage 2 negative).

d. Use $a = \dfrac{V}{40}$ and the time $260 - 2V$ for the constant speed stage. $a = \dfrac{32.3}{40} =$ **0.81 m/s^2**. Constant speed time $= 260 - 64.6 =$ **195.4 s**.

Answer to Question 20

a. Take upwards as positive for both particles; both have acceleration -9.8. Height of A: $80 - 4.9t^2$. Height of B: $ut - 4.9t^2$. They meet when these are equal: $80 = ut$, so $t = \dfrac{80}{u}$. (The $4.9t^2$ terms cancel because both objects accelerate equally.) So the meeting time does not depend on g.

b. Substitute $u = 40$ into $t = \dfrac{80}{u}$, then find the height of either particle. $t = \dfrac{80}{40} =$ **2 s**. Height $= 40 \times 2 - 4.9 \times 4 = 80 - 19.6 =$ **60.4 m**. They meet 2 s after release, 60.4 m above the ground.

c. Set the height of A at the meeting time equal to 20 m. Height $= 80 - 4.9\left(\dfrac{80}{u}\right)^2 = 20$, so $4.9 \times \dfrac{6400}{u^2} = 60$, $u^2 = \dfrac{31360}{60} = 522.7$, so $u =$ **22.9 m/s**. Check: $t = \dfrac{80}{22.9} = 3.5$ s and $80 - 4.9 \times 3.5^2 = 20$ ✓.