Test answers - Integration

Year: Year 12 · Duration: 60 minutes · /20

Answer to Question 1 (4 marks)

Write each term as a power of x and integrate term by term; in (b) expand the bracket first.

(a) $2x^4 + 3x^{-1} + \frac{2}{3}x^{3/2} + c$ (2). (b) Expand $x^2 - 6x + 9$: $\frac{x^3}{3} - 3x^2 + 9x + c$ (2).

Check (a) by differentiating: $8x^3 - 3x^{-2} + x^{1/2}$ ✓. Do not forget the constant c in each answer.

Answer (b) can be checked by differentiating: $x^2 - 6x + 9 = (x-3)^2$ ✓.

In (a), $-3x^{-2}$ integrates to $-3 \times \frac{x^{-1}}{-1} = 3x^{-1}$, so the sign changes.

Answer to Question 2 (4 marks)

Integrate the gradient function term by term, then substitute the point $(2, 5)$ to find the constant.

$y = x^3 - 4x^2 + x + c$ (2). $8 - 16 + 2 + c = 5$, so $c = 11$ (1). $y = x^3 - 4x^2 + x + 11$ (1).

Check: at $x = 2$, $8 - 16 + 2 + 11 = 5$ ✓. So the curve is $y = x^3 - 4x^2 + x + 11$.

Check by differentiating: $\frac{dy}{dx} = 3x^2 - 8x + 1$ ✓.

Mark scheme: 2 for the integral (1 if one term is wrong), 1 for using $(2, 5)$, 1 for the final equation. Without the constant the curve would pass through $(2, -6)$, not $(2, 5)$.

Answer to Question 3 (4 marks)

Write $\frac{6}{\sqrt{x}} = 6x^{-1/2}$, which integrates to $12x^{1/2}$; then substitute the limits 4 and 1.

$\left[12x^{1/2} + \frac{x^2}{2}\right]_1^4$ (2) $= (24 + 8) - (12 + 0.5)$ (1) $= 19.5$ (1).

So the value of the integral is 19.5.

Note that $\sqrt{4} = 2$, so $12x^{1/2}$ equals 24 at $x = 4$ and 12 at $x = 1$.

A frequent error is integrating $6x^{-1/2}$ to $3x^{1/2}$ by multiplying by the power instead of dividing by it. The numerical integration function of a calculator also gives 19.5.

Answer to Question 4 (4 marks)

Factorise to find the roots, which are the limits; the curve is below the axis between them.

Roots $x = 2, 4$ (1). $\int_2^4 (x^2 - 6x + 8)dx = \left[\frac{x^3}{3} - 3x^2 + 8x\right]_2^4$ (1)
$= \left(\frac{64}{3} - 48 + 32\right) - \left(\frac{8}{3} - 12 + 16\right) = \frac{16}{3} - \frac{20}{3} = -\frac{4}{3}$ (1). Area $= \frac{4}{3}$ (1).

The integral is negative because the region is below the x-axis, so the area is its modulus, $\frac{4}{3}$ square units.

By symmetry, the region lies between $x = 2$ and $x = 4$, with the lowest point of the curve at $(3, -1)$.

Answer to Question 5 (4 marks)

a. Compare the heights of the line and the curve at a point between the intersections. At $x = 2$: line 2, curve 1 (1) Since $2 > 1$, the line is above the curve on the whole interval.

b. Integrate (line $-$ curve) from 0 to 4. $\int_0^4 \left(x - \frac{1}{4}x^2\right)dx = \left[\frac{x^2}{2} - \frac{x^3}{12}\right]_0^4$ (1)
$= 8 - \frac{16}{3} = \frac{8}{3}$ (2 for evaluation) The area between the ramp and the line is $\frac{8}{3} \approx 2.67$ m^2.

Integrating curve minus line would give $-\frac{8}{3}$; an area is always positive.