Test answers - Indices, surds and quadratic functions

Year: Year 12 · Duration: 60 minutes · /20

Answer to Question 1 (5 marks)

a. Use $(ab)^n = a^n b^n$, so $(3x^2)^3 = 27x^6$, then divide the numbers and subtract the powers. $\frac{27x^6}{9x^{-1}} = 3x^{6-(-1)} = 3x^7$ (1 for $27x^6$, 1 for dividing, 1 for the answer) So the expression equals $3x^7$.

b. Rationalise the denominator using the conjugate $\sqrt{5}+1$; the denominator becomes $(\sqrt{5})^2 - 1^2 = 4$. Multiply by $\frac{\sqrt{5}+1}{\sqrt{5}+1}$: $\frac{10(\sqrt{5}+1)}{5-1}$ (1) $= \frac{5(\sqrt{5}+1)}{2}$ (1) Simplifying $\frac{10}{4} = \frac{5}{2}$ gives the required form.

In (a), a common error is $(3x^2)^3 = 3x^6$, forgetting to cube the 3.

Answer to Question 2 (4 marks)

a. Halve the coefficient of x to get $(x+5)^2$, then subtract $5^2 = 25$. $(x+5)^2 - 25 + 7 = (x+5)^2 - 18$ (1 for $a = 5$, 1 for $b = -18$) So $f(x) = (x+5)^2 - 18$.

b. The minimum occurs when the square is zero; then solve $(x+5)^2 = 18$ using $\sqrt{18} = 3\sqrt{2}$. Minimum $(-5, -18)$ (1). $(x+5)^2 = 18$, $x = -5 \pm 3\sqrt{2}$ (1) The roots are $x = -5 + 3\sqrt{2}$ and $x = -5 - 3\sqrt{2}$.

Check (b): $x \approx -0.757$ or $x \approx -9.243$, and both make $x^2 + 10x + 7$ zero. The exact surd form is required for the mark, so decimals alone score nothing.

Answer to Question 3 (4 marks)

a. Equal roots means $b^2 - 4ac = 0$ with $a = 1$, $b = k - 1$, $c = 4$. $(k-1)^2 - 16 = 0$ (1), $k - 1 = \pm 4$ (1), so $k = 5$ or $k = -3$ (1) Both values give equal roots.

b. With $k = 5$ the equation is a perfect square. $k = 5$: $x^2 + 4x + 4 = 0$, $x = -2$ (1) (With $k = -3$ the equation is $(x-2)^2 = 0$, so $x = 2$.)

Check $k = -3$: $x^2 - 4x + 4$ has discriminant $16 - 16 = 0$ ✓. Writing only $k - 1 = 4$ loses the negative square root and one value of k; remember $\pm$ when square-rooting.

Answer to Question 4 (3 marks)

$x^4 = (x^2)^2$, so substitute $y = x^2$ to obtain a quadratic in y.

Let $y = x^2$: $y^2 - 10y + 9 = 0$ (1), $(y-1)(y-9) = 0$ so $y = 1$ or 9 (1), so $x = \pm 1$ or ± 3 (1).

All four solutions are valid because 1 and 9 are positive, so each gives two square roots: $x = -3, -1, 1, 3$.

Check $x = 3$: $81 - 90 + 9 = 0$ ✓. Stopping at $y = 1$ or 9 without returning to x loses the final mark, and so does giving only the positive roots.

Answer to Question 5 (4 marks)

a. Half the perimeter is 50 cm, so the length is $50 - w$. Length $= 50 - w$ so $A = w(50 - w) = 50w - w^2$ (1) This is the required expression.

b. Take out the factor -1 and complete the square. $A = -(w - 25)^2 + 625$ (1), maximum 625 cm^2 when $w = 25$ (1) So the poster should be a 25 cm by 25 cm square.

c. Use the completed-square form and solve the inequality. $-(w - 25)^2 + 625 \geq 600$ gives $(w - 25)^2 \leq 25$, so $20 \leq w \leq 30$ (1) So the width must be between 20 cm and 30 cm.