Test answers - Hypothesis testing

Year: Year 12 · Duration: 60 minutes · /20

Answer to Question 1 (4 marks)

The claim is tested against a lower probability, so the test is one-tailed with the lower tail $P(X \leq 9)$.

$H_0 : p = 0.8$, $H_1 : p < 0.8$ (1). $X \sim B(15, 0.8)$ (1). $P(X \leq 9) = 0.0611$ (1) > 0.05, do not reject H_0; insufficient evidence that the trains are less punctual than claimed (1).

Although 9 out of 15 is only 60%, a result this low happens about 6% of the time when $p = 0.8$, which is not unusual enough at the 5% level.

Answer to Question 2 (4 marks)

For an upper-tail test the critical region is $X \geq c$; work down from the top using $P(X \geq c) = 1 - P(X \leq c - 1)$.

We need the smallest c with $P(X \geq c) \leq 0.05$ (1). $P(X \geq 12) = 0.0442 \leq 0.05$ and $P(X \geq 11) = 0.0978 > 0.05$ (2). Critical region $X \geq 12$, actual significance level 4.42% (1).

So any value of 12 or more leads to rejecting H_0, and the probability of wrongly doing so is 4.42%.

Answer to Question 3 (4 marks)

The question asks whether the proportion differs, so the test is two-tailed and each tail is allowed 2.5%. Under H_0, $X \sim B(20, 0.5)$ and 5 is in the lower tail.

$H_0 : p = 0.5$, $H_1 : p \neq 0.5$ (1). $P(X \leq 5) = 0.0207$ (1). $0.0207 < 0.025$, reject H_0 (1). Sufficient evidence that the proportion preferring A is not 50% (1).

The data suggest that fewer than half of customers prefer flavour A.

Alternatively use the critical region: $P(X \leq 5) = 0.0207$ but $P(X \leq 6) = 0.0577$, so the lower critical region is $X \leq 5$ and the observed value 5 lies in it. Comparing with 0.05 instead of 0.025 is a common mistake in two-tailed tests.

Answer to Question 4 (4 marks)

a. The significance level is the probability of rejecting H_0 when it is true. There is a 5% (or less) probability of observing a result this extreme if H_0 were true (2). So rejecting H_0 means the data would be unusual if H_0 were true.
b. A hypothesis test is based on probability, not certainty. The result could still happen by chance when H_0 is true (a Type I error); the test only gives evidence (2). So we say there is evidence for H_1, never that H_1 is proved.

Answer to Question 5 (4 marks)

a. The vaccine should reduce the infection rate, so the test is one-tailed (lower tail) with $X \sim B(20, 0.3)$. $H_0 : p = 0.3$, $H_1 : p < 0.3$ (1). $P(X \leq 2) = 0.0355 < 0.05$ (1), reject H_0: evidence that the vaccine reduces the infection rate (1) Only 2 out of 20 (10%) catching the virus is unlikely if the vaccine had no effect.

b. The critical region is the largest lower tail with probability at most 5%: $P(X \leq 2) = 0.0355$ but $P(X \leq 3) = 0.1071$. The critical region is $X \leq 2$, so the actual level is 3.55% (1) This is the probability of wrongly concluding that the vaccine works.

The conclusion in (a) must mention the vaccine to earn the final mark.