Worksheet - Hypothesis testing

Year: Year 12 · 20

Show your working.

Question 1

Question 1 – The language of hypothesis testing *(Section A · Skills practice)*

Ravi suspects a coin is biased towards heads. He tosses it 20 times and gets 15 heads. Let p be the probability of heads.

a. State suitable null and alternative hypotheses. [2 marks]
b. State the distribution of the number of heads X if the null hypothesis is true. [1 mark]
c. Find $P(X \geq 15)$. [2 marks]
d. Test at the 5% significance level and state your conclusion in context. [3 marks]

Question 2

Question 2 – A one-tailed test for a drug *(Section A · Skills practice)*

A drug is known to cure 40% of patients. A new drug is tested on 20 patients and 12 are cured. Test at the 5% level whether the new drug is more effective.

a. State the hypotheses. [2 marks]
b. Calculate the probability of 12 or more cures if $p = 0.4$. [3 marks]
c. State the conclusion in context. [3 marks]

Question 3

Question 3 – A two-tailed test *(Section A · Skills practice)*

A spinner is supposed to land on red with probability 0.25. In 40 spins it lands on red 17 times. Test at the 5% level whether the probability of red has changed.

a. State the hypotheses and explain why the test is two-tailed. [3 marks]
b. Find $P(X \geq 17)$ under H_0. [3 marks]
c. Compare with the correct significance level for each tail and state the conclusion. [4 marks]

Question 4

Question 4 – Finding a critical region (lower tail) *(Section A · Skills practice)*

$X \sim B(20, p)$. A test of $H_0 : p = 0.3$ against $H_1 : p$ is carried out at the 5% level.

r	0	1	2	3	4
$P(X \leq r)$	0.0008	0.0076	0.0355	0.1071	0.2375

a. Find the critical region. [3 marks]
b. State the actual significance level of the test. [1 mark]
c. If the observed value of X is 3, what is the conclusion? [2 marks]

Question 5

Question 5 – Finding a two-tailed critical region *(Section A · Skills practice)*

$X \sim B(20, 0.5)$. Find the critical region for a two-tailed test of $H_0 : p = 0.5$ at the 10% significance level, with at most 5% in each tail.

a. Find the lower tail. [3 marks]
b. Find the upper tail. [3 marks]
c. State the actual significance level. [2 marks]

Question 6

Question 6 – Writing the conclusion *(Section A · Skills practice)*

Rewrite each conclusion correctly in context.

a. "Reject H_0" in a test about a new fertiliser, where p is the proportion of plants that flower. [2 marks]
b. "Accept H_0" in a test about a bus company claiming 90% of buses are on time. [2 marks]
c. Why is it wrong to say that a test has "proved" the null hypothesis? [2 marks]

Question 7

Question 7 – Prizes in cereal boxes *(Section B · Real-life practice)*

A cereal company says 25% of its boxes contain a prize. A shopper buys 15 boxes and finds prizes in 7 of them. Test at 5% whether the proportion of prize boxes is greater than the company says.

a. State the hypotheses. [2 marks]
b. Find $P(X \geq 7)$ under H_0. [3 marks]
c. State the conclusion. [2 marks]
d. Find the critical region for the test and its actual significance level. [3 marks]

Question 8

Question 8 – Seed germination *(Section B · Real-life practice)*

A seed company claims that 80% of its seeds germinate. A gardener plants 25 seeds

and 16 germinate. She suspects the percentage is lower. Test at the 5% level.

a. State the hypotheses. [2 marks]
b. Find the probability of 16 or fewer germinating if the claim is true. [3 marks]
c. State your conclusion. [2 marks]
d. The gardener repeats the test and 17 germinate. What is the conclusion now? [3 marks]

Question 9

Question 9 – Is the die fair? *(Section B · Real-life practice)*

A fair die has probability $\frac{1}{6}$ of showing a six. A student rolls a die 30 times and gets only one six. She suspects the die rarely shows a six.

a. State the hypotheses. [2 marks]
b. Find $P(X \leq 1)$ under H_0 to 4 decimal places. [3 marks]
c. Test at the 5% level. [2 marks]
d. Explain why getting zero sixes would be stronger evidence. [1 mark]

Question 10

Question 10 – A new treatment *(Section B · Real-life practice)*

The recovery rate for a condition is 65%. A new treatment is tried on 12 patients and all 12 recover.

a. State the hypotheses. [2 marks]
b. Calculate the probability that all 12 recover if $p = 0.65$. [2 marks]
c. Conclude at the 1% level. [3 marks]
d. Another trial gives 11 recoveries out of 12. Is that significant at the 5% level? [3 marks]

Question 11

Question 11 – App crashes *(Section B · Real-life practice)*

Before an update, 10% of users reported a crash. After the update, 50 users are asked and 3 report a crash. The developers hope the crash rate has fallen.

a. State the hypotheses. [2 marks]
b. Find $P(X \leq 3)$ under H_0. [2 marks]
c. Test at the 5% level. [2 marks]
d. Find the critical region and actual significance level. [4 marks]

Question 12

Question 12 – A change in opinion *(Section B · Real-life practice)*

In a previous poll, half the voters in a town supported a plan. A new poll of 30 people finds that only 8 support it. The council asks whether support has changed.

a. State the hypotheses. [2 marks]
b. Find $P(X \leq 8)$ under H_0. [2 marks]
c. Carry out the test at the 5% level. [3 marks]
d. Give the conclusion in context. [2 marks]

Question 13

Question 13 – Defective components *(Section B · Real-life practice)*

A manufacturer says that only 2% of its components are defective. A buyer tests 40 components and finds 4 defective.

a. State the hypotheses. [2 marks]
b. Find $P(X \geq 4)$ under H_0. [3 marks]
c. Test at the 1% level and at the 5% level. [3 marks]
d. What would you advise the buyer? [1 mark]

Question 14

Question 14 – Has training helped? *(Section B · Real-life practice)*

A netball player scores 60% of her shots. After a training programme she scores 19 out of 25 shots. Test at the 5% level whether she has improved.

a. State the hypotheses. [2 marks]
b. Find $P(X \geq 19)$ under H_0. [3 marks]
c. State the conclusion. [2 marks]
d. How many successes out of 25 would have been needed to show improvement at the 5% level? [3 marks]

Question 15

Question 15 – A table of critical values *(Section C · Problem solving)*

A teacher claims that 30% of a school\'s students walk to school. A sample of 20 students is taken to test whether the proportion is lower. Here are cumulative probabilities for $X \sim B(20, 0.3)$.

r	0	1	2	3	4	5	6	7
$P(X \leq r)$	0.0008	0.0076	0.0355	0.1071	0.2375	0.4164	0.6080	0.7723

a. Find the critical region at the 5% level and the actual significance level. [3 marks]
b. Find the critical region at the 10% level. [2 marks]
c. The sample has 3 who walk. State the conclusions at 5% and 10%. [3 marks]
d. Explain why the actual significance level is less than the nominal one. [2 marks]

Question 16

Question 16 – Critical region with a larger sample *(Section C · Problem solving)*

A drug is known to be effective for 40% of patients. A two-tailed test at the 5% level (at most 2.5% in each tail) is carried out on a sample of 50 patients. Some probabilities for $X \sim B(50, 0.4)$ are shown.

r	12	13	14	15
$P(X \leq r)$	0.0133	0.0280	0.0540	0.0955
r	26	27	28	
$P(X \geq r)$	0.0573	0.0314	0.0160	

a. Find the lower critical value. [2 marks]
b. Find the upper critical value. [2 marks]
c. Write down the critical region and the actual significance level. [3 marks]
d. The sample gives 28 effective cases. State the conclusion. [2 marks]

Question 17

Question 17 – How good is the test? *(Section C · Problem solving)*

$X \sim B(20, p)$. A test of $H_0 : p = 0.2$ against $H_1 : p > 0.2$ uses the critical region $X \geq 8$.

a. Find the actual significance level. [2 marks]
b. The true value is actually $p = 0.4$. Find the probability that the test correctly rejects H_0. [3 marks]
c. Find the probability that the test fails to reject H_0 when $p = 0.4$. [1 mark]
d. Suggest two ways of making the test better at detecting $p = 0.4$. [2 marks]

Question 18

Question 18 – Planning how many trials to run *(Section C · Problem solving)*

To test whether a coin is biased towards heads, a student plans to toss it n times and reject $H_0 : p = 0.5$ in favour of $H_1 : p > 0.5$ if the results are extreme enough, at the 5% level.

n	5	6	7	8	9
$P(X \geq n-1) = \dfrac{n+1}{2^n}$	0.1875	0.1094	0.0625	0.0352	0.0195

a. Show that if all n tosses give heads, the probability under H_0 is 0.5^n and find the smallest n for which this is below 5%. [3 marks]
b. Show that $P(X \geq n-1) = \dfrac{n+1}{2^n}$. [3 marks]
c. Find the smallest n for which getting at least $n-1$ heads would be significant at 5%. [2 marks]

d. Comment on the usefulness of tosses with small n. [2 marks]

Question 19

Question 19 – Five classes, one claim *(Section C · Problem solving)*

A school claims that 70% of students prefer the new canteen menu. Five classes of 10 students each are surveyed independently, with the following numbers who prefer the new menu.

Class	A	B	C	D	E
Number preferring (out of 10)	8	6	9	4	7

a. Test the claim for class D at the 5% level (one-tailed, p). [4 marks]
b. Combine the classes into a single sample of 50 and test again. [5 marks]
c. Explain why the combined test is more reliable. [2 marks]

Question 20

Question 20 – Challenge: one-tailed versus two-tailed *(Section D · Challenge)*

A coin is tossed 30 times and lands heads 20 times.

a. Test at the 5% level whether the coin is biased towards heads (one-tailed). [4 marks]
b. Test at the 5% level whether the coin is biased (two-tailed). [4 marks]
c. Explain why the two conclusions differ and why the choice of H_1 must be made before looking at the data. [3 marks]