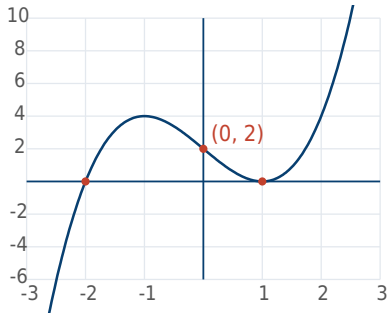
Test answers - Graphs and transformations

Year: Year 12 · Duration: 60 minutes · /20

Answer to Question 1 (4 marks)

Each factor gives a root; the squared factor $(x-1)^2$ gives a repeated root where the curve touches the axis.

Roots at $x=-2$ (crosses) and $x=1$ (touches, repeated) (2). y-intercept: $1\times 2=2$ at $(0,2)$ (1). Positive cubic: falls on the left, rises on the right, with a turning point on the axis at $(1,0)$ (1).



So the curve crosses the axis at $(-2,0)$, meets the y-axis at $(0,2)$ and touches the x-axis at $(1,0)$.

Answer to Question 2 (4 marks)

a. $f(x+1)$ moves the graph 1 unit left. $(2,-2)$ (1)
b. $3f(x)$ multiplies y by 3. $(3,-6)$ (1)
c. $f(x/3)$ multiplies x by 3. $(9,-2)$ (1)
d. Reflect in the x-axis first, so -2 becomes 2, then add 2. $(3,4)$ (1: reflect to 2 then add 2) So P maps to $(3,4)$.

In (c), the stretch with scale factor 3 parallel to the x-axis leaves $y=-2$ unchanged. In (d), doing the translation before the reflection would give $(3,0)$, which is wrong.

Answer to Question 3 (4 marks)

The graph is $y=\dfrac{1}{x}$ translated by $\begin{pmatrix}-2\\-3\end{pmatrix}$, so the asymptotes move to $x=-2$ and $y=-3$.

Asymptotes $x=-2$ and $y=-3$ (2). y-intercept $\frac{1}{2}-3=-\frac{5}{2}$ (1). x-intercept: $\dfrac{1}{x+2}=3$, $x=-\frac{5}{3}$ (1).

So the curve crosses the axes at $(0,-\frac{5}{2})$ and $(-\frac{5}{3},0)$.

Sketch: the curve has two branches. For $x > -2$ the branch lies above $y = -3$ and passes through both intercepts; for $x < -2$ it lies below $y = -3$. Marks: 1 for each asymptote correctly labelled, 1 for each intercept. Check: at $x = -\frac{5}{3}$, $\frac{1}{1/3} - 3 = 0$ ✓.

Answer to Question 4 (3 marks)

Inside the bracket $x - 5$ moves the graph 5 right; outside, $+1$ moves it up. Replacing x by $2x$ halves every x-coordinate.

(a) Translation by $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$ (2). (b) Stretch with scale factor $\frac{1}{2}$ parallel to the x-axis (1).

A translation needs its vector; a stretch needs its scale factor and direction.

Writing ‘move 5 right and 1 up’ scores the marks only if the word translation or the vector is given. In (b), a stretch of scale factor 2 is the most common wrong answer.

Answer to Question 5 (5 marks)

a. A positive cubic with a repeated root at 0 and a single root at 3. Roots at 0 (touching) and 3; max $(0, 0)$; min $(2, -4)$ (2)
b. Count the intersections of the line $y = k$ with the curve, using the turning points. $k = -2$: 3 solutions (1). $k = -4$: 2 solutions (1) For $k = -4$ the line touches the minimum, giving a repeated root.
c. Expand $(x - 2)^2$ first, then multiply by $(x + 1)$.
$(x + 1)(x^2 - 4x + 4) = x^3 - 3x^2 + 4$ (1) This confirms the result.