Lesson - Forces and Newton's laws

Year: Year 12

Mechanics models real objects as **particles** (all the mass at a point) and forces as vectors. This chapter shows how to find the resultant force, apply Newton's second law and solve connected-particle problems. A clear force diagram is the first mark in every question.

Starter activity

You push a heavy shopping trolley and it slowly speeds up. You push the same trolley when it is full and it speeds up more slowly. List the quantities that change and the quantities that stay the same. Which would you need to know to predict how quickly the trolley speeds up? Now imagine pushing it up a slope. What new force needs to be considered, and in which direction does it act? Draw a force diagram for each situation.

Forces and units

Definition: force

A **force** is a vector measured in newtons (N). Common forces: **weight** $W = mg$ acting down ($g = 9.8$ m s^{-2}), **normal reaction** R perpendicular to a surface, **tension** T in a string or rope (pulls along the string; the same size throughout a light string over a smooth pulley), **thrust** and **resistance**. A 5 kg mass has weight $5 \times 9.8 = 49$ N.

Newton's laws

Theorem: Newton's laws

1. **First law.** A particle stays at rest or moves with constant velocity unless a resultant force acts. If the forces balance (resultant $= 0$) the particle is in **equilibrium**.
2. **Second law.** The resultant force equals mass times acceleration: $\mathbf{F} = m\mathbf{a}$, and in one direction $F = ma$.
3. **Third law.** If A exerts a force on B, then B exerts a force of equal size and opposite direction on A.

Worked example 1

An 800 kg car has driving force 2400 N and resistance 800 N. Find its acceleration and its speed and distance after 10 s from rest.

Resultant $= 2400 - 800 = 1600$ N, so $a = \dfrac{1600}{800} = 2$ m/s^2. After 10 s $v = 20$ m/s and

the distance is $\frac{1}{2}(2)(100) = 100$ m.

Resolving forces

Rule: components of a force

A force F at angle θ to the horizontal has horizontal component $F\cos\theta$ and vertical component $F\sin\theta$. In equilibrium the components in each direction sum to zero. For forces at right angles the resultant has magnitude $\sqrt{F_1^2 + F_2^2}$; forces in $\mathbf{i}, \mathbf{j}$ form simply add.

Worked example 2

Forces of 6 N and 8 N act at right angles. Find the resultant.

$\sqrt{36 + 64} = 10$ N, at $\tan^{-1}\dfrac{8}{6} = 53.1^\circ$ to the 6 N force.

Worked example 3

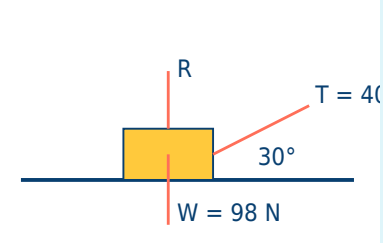
Forces $3\mathbf{i} + 4\mathbf{j}$ and $-5\mathbf{i} + 2\mathbf{j}$ act on a particle in equilibrium with a third force $\mathbf{F}$. Find $\mathbf{F}$.

The forces sum to zero: $\mathbf{F} = -(3\mathbf{i} + 4\mathbf{j}) - (-5\mathbf{i} + 2\mathbf{j}) = 2\mathbf{i} - 6\mathbf{j}$.

Worked example 4

A 10 kg crate is pulled along a smooth floor by a rope with tension 40 N at 30° above the horizontal. Find a and R.

Horizontal: $40\cos 30^\circ = 10a$, so $a = 3.46$ m/s^2. Vertical (no vertical acceleration): $R + 40\sin 30^\circ = 98$, so $R = 78$ N.



Watch out

Check which angle you were given before choosing sine or cosine. Weight is mg, not m. Do not forget the normal reaction or tension on the diagram.

Motion on a slope

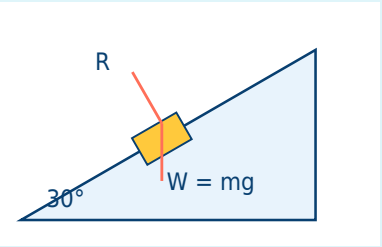
Rule: smooth slope at angle α

Resolve the weight: down the slope $mg\sin\alpha$ and perpendicular to the slope $mg\cos\alpha$. So $R = mg\cos\alpha$ (not mg) and the acceleration down the slope is $g\sin\alpha$.

Worked example 5

A 5 kg block rests on a smooth 30° slope. Find a and R.

$a = 9.8\sin 30^\circ = 4.9$ m/s^2 and $R = 49\cos 30^\circ = 42.4$ N.



Connected particles

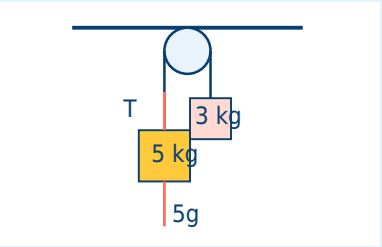
Method: connected particles

1. Draw a force diagram for *each* particle.
2. A light inextensible string gives the same acceleration size; over a smooth pulley the tension is the same at both ends.
3. Write $F = ma$ for each particle, with the direction of acceleration positive.
4. Solve the equations together.

Worked example 6

Particles of 5 kg and 3 kg are connected by a string over a smooth pulley. Find a and T.

Heavier particle (down): $5g - T = 5a$. Lighter (up): $T - 3g = 3a$. Adding: $2g = 8a$, so $a = \dfrac{2 \times 9.8}{8} = 2.45$ m/s^2. Then $T = 3(g + a) = 36.75$ N.



Worked example 7

A 1200 kg car tows a 600 kg caravan. Driving force 3600 N, resistances 500 N (car) and 300 N (caravan). Find a and the tension.

Whole system: $3600 - 800 = 1800a$, so $a = 1.56$ m/s^2. Caravan alone: $T - 300 = 600a$, giving $T = 1233$ N.

Worked example 8

A 60 kg person stands in a lift. Find the reaction when it accelerates upwards at 1.2 m/s^2 and downwards at 1.2 m/s^2.

Up: $R - 588 = 60 \times 1.2$, $R = 660$ N (heavier). Down: $588 - R = 72$, $R = 516$ N.

Watch out

Write $F = ma$ with the sign for the direction of motion. Always resolve in two perpendicular directions and include every force.

Worked example 9: A force at an angle on a rough surface

A box of mass 10 kg is pulled along rough horizontal ground by a rope at 25° above the horizontal with tension 60 N. The coefficient of friction is 0.2. Find the acceleration.

Vertically: $R + 60\sin 25^\circ = 10g$, so $R = 98 - 25.36 = 72.64$ N. Friction $F = 0.2 \times 72.64 = 14.53$ N. Horizontally: $60\cos 25^\circ - 14.53 = 10a$, so $54.38 - 14.53 = 39.85 = 10a$ and $a = 3.99$ m/s^2. The rope lifts the box a little, which reduces the normal reaction and therefore the friction. Always resolve vertically first to find R, then use $F = \mu R$.

Worked example 10: Resolving on a slope

A 12 kg crate is held at rest on a smooth slope of 35° by a force P acting parallel to the slope up the slope. Find P and the normal reaction.

Parallel to the slope: $P = mg\sin 35^\circ = 12 \times 9.8 \times 0.5736 = 67.5$ N. Perpendicular: $R = mg\cos 35^\circ = 12 \times 9.8 \times 0.8192 = 96.3$ N. If P is removed the crate accelerates down the slope at $g\sin 35^\circ = 5.62$ m/s^2 and R is unchanged. Notice that the mass cancels in the acceleration, so every object slides down a smooth slope at the same rate.

Worked example 11: Newton's third law

A 70 kg skater pushes off a wall with a force of 140 N for 0.5 s. Find the speed gained and the force of the skater on the wall.

By Newton\'s third law the wall pushes the skater with 140 N, and the skater pushes the wall with an equal and opposite force of 140 N. The skater\'s acceleration is

$\frac{140}{70} = 2$ m/s^2, so after 0.5 s the speed is $2 \times 0.5 = 1$ m/s. The two forces in a third-law pair act on different bodies, which is why they do not cancel each other in the equation of motion of the skater.

Worked example 12: A pulley with a particle on a smooth slope

A 5 kg particle on a smooth slope of 30° is attached by a string over a pulley at the top to a 3 kg particle hanging freely. Find the acceleration.

Take the direction of motion with the 5 kg particle moving down the slope if $5g\sin 30^\circ > 3g$. Here $5g\sin 30^\circ = 24.5$ N and $3g = 29.4$ N, so the hanging particle goes down and the particle on the slope goes up it. Equations: $3g - T = 3a$ and $T - 5g\sin 30^\circ = 5a$. Add: $29.4 - 24.5 = 8a$, so $a = 0.6125$ m/s^2. Then $T = 3(9.8 - 0.6125) = 27.56$ N. Check the tension in the other equation: $27.56 - 24.5 = 3.06 = 5 \times 0.6125$ ✓.

Worked example 13: Tension in a rope hauling a load

A crane lifts a 250 kg load with an acceleration of 0.4 m/s^2 upwards. Find the tension in the cable and the cable tension when the load moves at constant speed.

$T - mg = ma$, so $T = 250(9.8 + 0.4) = 2550$ N. At constant speed $a = 0$, so $T = mg = 2450$ N. The tension is larger than the weight when the load speeds up and smaller than the weight when it slows down while rising: for a deceleration of 0.4 m/s^2 the tension is $250(9.8 - 0.4) = 2350$ N.

Method: a Newton's second law problem

1. Draw a clear force diagram showing the weight, normal reaction, friction, tensions and applied forces; mark the acceleration.
2. Choose directions and resolve forces parallel and perpendicular to the motion.
3. Write $F = ma$ for each object in the direction of motion (and R or tension equations perpendicular to it).
4. For connected particles write one equation for each particle and solve them together.
5. Check the answer is reasonable and give the units.

Watch out: common mistakes with forces

- Confusing mass (kg) with weight (N); the weight is mg.
- Forgetting that on a slope the normal reaction is $mg\cos\theta$, not mg.
- Using $F = \mu mg$ when the friction is not limiting, or when the normal reaction is not mg.
- Giving the tension on both sides of a pulley different values when the pulley is

smooth and light: the tension is the same.
- Not stating the direction of the acceleration or of the resultant force.

Note: Newton's three laws

The first law says that a body stays at rest or moves at constant velocity unless a resultant force acts on it, so equilibrium means the resultant is zero. The second law, $\mathbf{F} = m\mathbf{a}$, links the resultant force to the acceleration and is the basis of almost every calculation in this chapter. The third law says that forces come in equal and opposite pairs acting on different bodies, such as the pull of the rope on the box and the pull of the box on the rope. When you draw a force diagram, include only the forces acting on the object you are analysing.

Did you know?

Isaac Newton published his three laws of motion in 1687 in the *Principia Mathematica*. The newton, the unit of force, is the force needed to give a 1 kg mass an acceleration of 1 m/s^2: about the weight of a small apple. Engineers use Newton\'s laws to design lifts, bridges and rockets. A rocket works through the third law: it pushes hot gas backwards and the gas pushes the rocket forwards.

Otto's tip

"Draw, label, resolve, then $F = ma$." The direction of acceleration is the positive direction.