Answers - Forces and Newton's laws

Year: Year 12

Answer to Question 1

a. Weight is mass times the acceleration due to gravity. $W = mg = 6.5 \times 9.8 =$ **63.7 N**. The parcel weighs 63.7 N.

b. Rearrange $W = mg$ to make m the subject. $m = \dfrac{W}{g} = \dfrac{245}{9.8} =$ **25 kg**. The crate has a mass of 25 kg.

c. On a smooth road there is no resistance, so the resultant force is the driving force; use Newton's second law. $F = ma = 1200 \times 1.5 =$ **1 800 N**. A force of 1 800 N is needed.

d. Force is measured in newtons (N); $1\ \text{N} = 1\ \text{kg m/s}^2$. Mass is the amount of matter (kg) and does not change; weight is the gravitational force on the mass (N) and depends on g.

Answer to Question 2

a. The forces are perpendicular, so use Pythagoras for the magnitude and trigonometry for the direction. Magnitude $\sqrt{36 + 64} =$ **10 N**. Direction $\tan^{-1}\left(\frac{8}{6}\right) =$ **53.1**$^\circ$ north of east. The resultant is 10 N on a bearing of about 037.

b. Again the forces are at right angles, so the magnitude comes from Pythagoras. $\sqrt{144 + 25} =$ **13 N**. The angle with the 12 N force is $\tan^{-1}\frac{5}{12} = 22.6^\circ$.

c. Add the $\mathbf{i}$ components and the $\mathbf{j}$ components separately. Sum $= 2\mathbf{i} + 6\mathbf{j}$ N. Magnitude $\sqrt{4 + 36} = \sqrt{40} =$ **6.32 N**. Angle with $\mathbf{i}$: $\tan^{-1} 3 = 71.6^\circ$. The resultant is 6.32 N at 71.6° to the $\mathbf{i}$ direction.

Answer to Question 3

a. The resultant force is the pull minus the resistance; then apply $F = ma$. $F = ma$: $50 - 18 = 8a$, so $a =$ **4 m/s**^{2}. The box accelerates at 4 m/s^2.

b. Use $v = u + at$ with $u = 0$. $v = at = 4 \times 5 =$ **20 m/s**. After 5 s the speed is 20 m/s.

c. Use $s = ut + \frac{1}{2}at^2$ with $u = 0$. $s = \frac{1}{2}at^2 = \frac{1}{2} \times 4 \times 25 =$ **50 m**. The box travels 50 m in 5 s.

d. Once the pull is removed, only the 18 N resistance acts, so the box decelerates; use $v = u + at$ with $v = 0$. Deceleration $= \dfrac{18}{8} = 2.25$ m/s^2. $t = \dfrac{20}{2.25} =$ **8.9 s**. The box stops about 8.9 s after the pull is removed.

Answer to Question 4

a. The horizontal component uses cosine and the vertical component uses sine of the angle with the horizontal. Horizontal $20\cos 30^\circ =$ **17.3 N**; vertical $20\sin 30^\circ =$ **10 N** (up).
b. Use the same method; the vertical component points downwards. Horizontal $15\cos 40^\circ =$ **11.5 N**; vertical $15\sin 40^\circ =$ **9.64 N** (down).
c. The force makes 20° with the horizontal ground, so resolve with $\cos 20^\circ$ and $\sin 20^\circ$. Horizontal $50\cos 20^\circ =$ **47.0 N**; vertical $50\sin 20^\circ =$ **17.1 N**.
d. Add the two horizontal components because they act in the same direction. $17.32 + 11.49 =$ **28.8 N**. The total horizontal force is 28.8 N.

Answer to Question 5

a. Equilibrium means the resultant is zero: $\mathbf{F}_3 = -(\mathbf{F}_1 + \mathbf{F}_2) = -(2\mathbf{i} + 6\mathbf{j}) = -2\mathbf{i} - 6\mathbf{j}$ **N**. Check: $(5 - 3 - 2)\mathbf{i} + (2 + 4 - 6)\mathbf{j} = \mathbf{0}$ ✓.
b. Use Pythagoras for the magnitude and $\tan$ for the angle. $|\mathbf{F}_3| = \sqrt{4 + 36} =$ **6.32 N**. The force points down-left; angle below the negative x-axis $= \tan^{-1}\left(\frac{6}{2}\right) = 71.6^\circ$.
c. Once $\mathbf{F}_3$ is removed, the resultant is equal and opposite to $\mathbf{F}_3$; apply $\mathbf{F} = m\mathbf{a}$. The resultant of the other two is $2\mathbf{i} + 6\mathbf{j}$, magnitude 6.32 N. $a = \dfrac{6.32}{4} =$ **1.58 m/s**^{2} in the direction of $2\mathbf{i} + 6\mathbf{j}$.

Answer to Question 6

a. On a smooth slope only two forces act on the box. Weight $mg = 49$ N vertically down; normal reaction R perpendicular to the slope. No friction (smooth). The weight is $5 \times 9.8 = 49$ N.
b. There is no acceleration perpendicular to the slope, so the forces balance in that direction. Perpendicular to the slope: $R = mg\cos 30^\circ = 49 \times 0.866 =$ **42.4 N**. The normal reaction is 42.4 N.
c. The only force along the slope is the component of the weight, $mg\sin 30^\circ$. Along the slope: $mg\sin 30^\circ = ma$, so $a = g\sin 30^\circ =$ **4.9 m/s**^{2}. The acceleration does not depend on the mass of the box.
d. Use $v = u + at$ with $u = 0$. $v = 4.9 \times 3 =$ **14.7 m/s**. After 3 s the box moves at 14.7 m/s.

Answer to Question 7

a. Take upwards as positive; the resultant force is $R - mg$. $R - mg = ma$: $R = 60(9.8 + 1.2) =$ **660 N**. The floor pushes with 660 N.

b. Take downwards as positive; the resultant force is $mg - R$. $mg - R = ma$: $R = 60(9.8 - 0.8) =$ **540 N**. The floor pushes with 540 N.
c. Constant speed means zero acceleration, so the forces balance. No acceleration: $R = mg =$ **588 N**. The floor pushes with 588 N.
d. Accelerating up the floor pushes harder (you feel heavier, 660 N); accelerating down you feel lighter (540 N); at constant speed you feel your normal weight.

Answer to Question 8

a. Treat the car and caravan as one body: the tension is internal and cancels. Whole system: $3500 - 600 - 400 = 2000a$, so $a =$ **1.25 m/s^2**. The acceleration is 1.25 m/s^2.
b. Now consider the caravan alone: the only horizontal forces on it are the tension and its resistance. Caravan alone: $T - 400 = 800 \times 1.25$, so $T =$ **1 400 N**. The tension in the tow bar is 1 400 N.
c. Take the direction of motion as positive and consider the whole system first. Forces opposing motion: $600 + 400 + 2000 = 3000$ N, so $a = \dfrac{-3000}{2000} = -1.5$ m/s^2 (deceleration 1.5). Caravan: With tension T forwards on the caravan: $T - 400 = 800 \times (-1.5)$, so $T = -800$ N, meaning the bar pushes (the tow bar is in thrust, not tension): **800 N thrust**.

Answer to Question 9

a. The heavier particle moves down and the lighter one moves up, with the same acceleration a because the string is inextensible. Heavier particle (down): $5g - T = 5a$. Lighter particle (up): $T - 3g = 3a$.
b. Adding the equations eliminates T. Add: $2g = 8a$, $a = \dfrac{2 \times 9.8}{8} =$ **2.45 m/s^2**. The particles accelerate at 2.45 m/s^2.
c. Substitute a into the equation for the lighter particle. $T = 3(g + a) = 3 \times 12.25 =$ **36.75 N**. The tension is 36.75 N.
d. Use $v = at$ and $s = \frac{1}{2}at^2$ from rest. $v = 2.45 \times 2 =$ **4.9 m/s**; distance $= \frac{1}{2} \times 2.45 \times 4 =$ **4.9 m**. Each particle moves 4.9 m in 2 s and reaches 4.9 m/s.

Answer to Question 10

a. On horizontal ground the normal reaction balances the weight. $R = mg = 196$ N. $F_{max} = \mu R = 0.3 \times 196 =$ **58.8 N**. The maximum friction is 58.8 N.
b. Compare the push with the maximum available friction. 50 N is less than 58.8 N, so friction balances the push: the box **does not move**.
c. Friction is now limiting and equal to 58.8 N; apply $F = ma$ to the resultant. $80 - 58.8 = 20a$, so $a =$ **1.06 m/s^2**. The box accelerates at 1.06 m/s^2.

d. Use $s = \frac{1}{2}at^2$ from rest. $5 = \frac{1}{2} \times 1.06t^2$, $t^2 = 9.43$, $t =$ **3.07 s**. The box takes about 3.07 s to travel 5 m.

Answer to Question 11

a. Each cable has vertical component $T\sin\theta$; in equilibrium the upward forces balance the 200 N weight. Resolving vertically: the vertical components of both tensions balance the weight, $2T\sin\theta = 200$.

b. Substitute $\sin 30^\circ = \frac{1}{2}$. $T = \dfrac{200}{2\sin 30^\circ} =$ **200 N**.

c. Substitute $\sin 20^\circ = 0.342$. $T = \dfrac{200}{2\sin 20^\circ} = \dfrac{100}{0.342} =$ **292.4 N**. The tension rises to 292.4 N.

d. $\sin\theta \leq 1$, so $T = \dfrac{100}{\sin\theta} \geq 100$ N. As $\theta \to 0$ the vertical component of each tension shrinks, so a bigger tension is needed. This is why cables are never pulled completely flat.

Answer to Question 12

a. Take upwards as positive; the resultant force is $T - mg$. $T - 250g = 250 \times 0.4$, so $T = 2450 + 100 =$ **2 550 N**. The tension is 2 550 N.

b. At constant speed the acceleration is zero, so the forces balance. $T = mg =$ **2 450 N**. The tension is 2 450 N.

c. Use $v^2 = u^2 + 2as$ with $u = 2$, $v = 0$, $s = 5$, then Newton's second law. $0 = 4 - 2a \times 5$, so the deceleration is $a = 0.4$ m/s^2. Then $250g - T = 250 \times 0.4$ gives $T = 2450 - 100 =$ **2 350 N**. The deceleration is 0.4 m/s^2 and the tension is 2 350 N.

d. Apply Newton's second law with $T = 2600$ N. $2600 - 2450 = 250a$, so $a =$ **0.6 m/s^2**. The greatest safe acceleration is 0.6 m/s^2.

Answer to Question 13

a. By symmetry the sideways components cancel and only the components along the bisector remain. The resultant lies along the bisector, at 30° to each rope: $2 \times 5\cos 30^\circ =$ **8.66 kN**. The combined pull is 8.66 kN.

b. Subtract the water resistance from the forward pull. $8.66 - 6 =$ **2.66 kN**. The net force is 2.66 kN forwards.

c. Convert to SI units (2 660 N and 40 000 kg) and use $a = \dfrac{F}{m}$. $a = \dfrac{2660}{40000} =$ **0.0665 m/s^2**. The ship accelerates at about 0.0665 m/s^2.

d. Each rope now makes 45° with the direction of motion. $2 \times 5\cos 45^\circ = 7.07$ kN, smaller than 8.66 kN. Pulling at a wider angle wastes some of the force sideways.

The net force would only be 1.07 kN.

Answer to Question 14

a. Use $v^2 = u^2 - 2as$ with $u = 25$, $v = 0$, $s = 50$. $0 = 625 - 2a \times 50$, so $a =$ **6.25 m/s^2**. The deceleration is 6.25 m/s^2.
b. Use Newton's second law. $F = ma = 900 \times 6.25 =$ **5 625 N**. The braking force is 5 625 N.
c. Use $v = u - at$ with $v = 0$. $t = \dfrac{25}{6.25} =$ **4 s**. The car takes 4 s to stop.
d. Find the new deceleration from $F = ma$, then use $v^2 = u^2 - 2as$. $a = \dfrac{3600}{900} = 4$ m/s^2. $s = \dfrac{625}{2 \times 4} =$ **78.1 m**. On the wet road the stopping distance increases to about 78.1 m.

Answer to Question 15

a. Treat the whole train as one body: the coupling forces are internal. Total mass 70 t = 70 000 kg, total resistance 35 000 N. $60000 - 35000 = 70000a$, $a =$ **0.357 m/s^2**. The train accelerates at 0.357 m/s^2.
b. Consider the three trucks together: the only external horizontal forces are the coupling tension and their 15 000 N resistance. Trucks 1-3 (30 000 kg): $T_1 - 15000 = 30000 \times 0.357$, so $T_1 = 15000 + 10714 =$ **25 714 N**. The tension is about 25 700 N.
c. Consider truck 3 alone. Truck 3 alone: $T_3 - 5000 = 10000 \times 0.357$, $T_3 =$ **8 571 N**. The tension in the last coupling is about 8 570 N.
d. Deceleration of the whole train: $\dfrac{20000 + 35000}{70000} = 0.786$ m/s^2. Trucks 1-3: the trucks need a retarding force of $30000 \times 0.786 = 23\,571$ N but resistance gives only 15 000 N, so the coupling must push backwards on the trucks with $23\,571 - 15\,000 = 8\,571$ N. **Thrust in the coupling = 8 571 N**.

Answer to Question 16

a. Take the direction of motion as positive for each particle; the tension is the same throughout the string. B: $6g - T = 6a$. A: $T - F = 4a$ where $F = 0.25 \times 4g = 9.8$ N.
b. Add the two equations to eliminate T. Add: $6g - 9.8 = 10a$, so $a = \dfrac{58.8 - 9.8}{10} =$ **4.9 m/s^2**. $T = 6g - 6a = 58.8 - 29.4 =$ **29.4 N**. The acceleration is 4.9 m/s^2 and the tension is 29.4 N.
c. Use $v^2 = u^2 + 2as$ with $u = 0$, $s = 1.2$. $v^2 = 2 \times 4.9 \times 1.2 = 11.76$, so $v =$ **3.43**

m/s. B hits the floor at 3.43 m/s.

d. Once the string is slack, only friction acts horizontally on A. Deceleration of A $= \frac{9.8}{4} = 2.45$ m/s^2. Distance $= \frac{v^2}{2a} = \frac{11.76}{4.9} =$ **2.4 m**. A slides another 2.4 m before stopping.

Answer to Question 17

a. In equilibrium the horizontal components of the tensions balance, and the vertical components support the weight $50g = 490$ N. Horizontally: $T_1 \cos 40^\circ = T_2 \cos 25^\circ$. Vertically: $T_1 \sin 40^\circ + T_2 \sin 25^\circ = 490$.

b. From the first, $T_2 = T_1 \frac{\cos 40^\circ}{\cos 25^\circ}$. Substitute: $T_1 \left[\sin 40^\circ + \cos 40^\circ \tan 25^\circ\right] = 490$. The bracket equals $\frac{\sin(40^\circ + 25^\circ)}{\cos 25^\circ} = \frac{\sin 65^\circ}{\cos 25^\circ} = 1$ (because $\sin 65^\circ = \cos 25^\circ$). So $T_1 =$ **490 N**.

c. Use the horizontal equation. $T_2 = 490 \times \frac{0.7660}{0.9063} =$ **414 N**. The tension in rope 2 is 414 N.

d. Rope 1 carries 490 N, the larger tension, and is steeper so it takes more of the weight.

Answer to Question 18

a. Resolve perpendicular to the slope, then use $F_{max} = \mu R$. $R = 8g \cos 30^\circ = 67.9$ N. $F_{max} = 0.4 \times 67.9 =$ **27.2 N**. Maximum friction is 27.2 N.

b. Compare the component of the weight down the slope with the maximum friction. Component of weight down the slope $= 8g \sin 30^\circ = 39.2$ N, which exceeds 27.2 N, so the block **slides**. $8a = 39.2 - 27.2 = 12.0$, $a =$ **1.51 m/s^2**. The block slides down with acceleration 1.51 m/s^2.

c. When the block moves up, both the weight component and friction act down the slope. $80 - 39.2 - 27.2 = 8a$, so $a = \frac{13.64}{8} =$ **1.71 m/s^2**. The block accelerates up the slope at 1.71 m/s^2.

d. On the point of moving up, the tension balances the weight component plus limiting friction. $T = 39.2 + 27.2 =$ **66.4 N**. The smallest tension is 66.4 N.

Answer to Question 19

a. Take upwards as positive and apply $T - mg = ma$ to the lift (total 800 kg) in each phase. $T - mg = ma$: phase 1: $800(9.8 + 1.5) =$ **9 040 N**; phase 2: $800 \times 9.8 =$ **7 840 N**; phase 3: $800(9.8 - 1.2) =$ **6 880 N**.

b. Apply $R - mg = ma$ to the 70 kg passenger with the same accelerations. Phase

1: $70(9.8 + 1.5) =$ **791 N**; phase 2: **686 N**; phase 3: $70(9.8 - 1.2) =$ **602 N**. The passenger feels heaviest in phase 1 and lightest in phase 3.

c. Phase 1, when the tension is greatest. The tension of 9 040 N is the largest of the three phases.

Answer to Question 20

a. In equilibrium the sum of the $\mathbf{i}$ components and the sum of the $\mathbf{j}$ components must both be zero. $\mathbf{i}$: $3k - 4 + 1 = 0$ gives $k = 1$. $\mathbf{j}$: $2 + (k + 1) - 4 = 0$ gives $k = 1$. Both agree: $k = 1$.

b. With $\mathbf{F}_3$ removed, the resultant is $\mathbf{F}_1 + \mathbf{F}_2$; use $\mathbf{F} = m\mathbf{a}$.
$\mathbf{F}_1 + \mathbf{F}_2 = (3 - 4)\mathbf{i} + (2 + 2)\mathbf{j} = -\mathbf{i} + 4\mathbf{j}$ N. $\mathbf{a} = \frac{1}{2}(-\mathbf{i} + 4\mathbf{j}) = (-0.5\mathbf{i} + 2\mathbf{j})$ **m/s**^{2}.
Magnitude $\sqrt{0.25 + 4} =$ **2.06 m/s**^{2}.

c. Sketch the vector $-0.5\mathbf{i} + 2\mathbf{j}$: it points left and up. The acceleration is in the second quadrant: angle $= 180^\circ - \tan^{-1}\left(\frac{2}{0.5}\right) = 180^\circ - 76.0^\circ =$ **104.0**$^\circ$ from the positive x-axis. The acceleration makes an angle of 104.0° with the positive x-axis.