Worksheet - Vectors

Year: Year 11 · 20

Show your working.

Question 1

Question 1 – Column vector arithmetic *(Section A · Skills practice)*

$\mathbf{a} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$. Work out each of the following and write your answer as a column vector.

a. $\mathbf{a} + \mathbf{b}$ [1 mark]
b. $\mathbf{a} - \mathbf{b}$ [1 mark]
c. $3\mathbf{c}$ [1 mark]
d. $2\mathbf{a} + 3\mathbf{b}$ [2 marks]
e. $\mathbf{a} - 2\mathbf{c}$ [2 marks]

Question 2

Question 2 – The size of a vector *(Section A · Skills practice)*

The magnitude (length) of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ is $\sqrt{x^2 + y^2}$. Find the magnitude of each vector. Give non-integer answers to 2 decimal places.

a. $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$
b. $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$
c. $\begin{pmatrix} -8 \\ 6 \end{pmatrix}$
d. $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$
e. $\begin{pmatrix} -4 \\ -7 \end{pmatrix}$

Question 3

Question 3 – Vectors between points *(Section A · Skills practice)*

Three points have coordinates $A(1, 2)$, $B(5, 5)$ and $C(-2, 4)$.

a. Write down the column vector $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$. [1 mark]

b. Work out and . [2 marks]

c. Work out . What do you notice? [2 marks]

d. Find the length of AB. [1 mark]

Question 4

Question 4 – Parallel vectors *(Section A · Skills practice)*

Here are four vectors: $\mathbf{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$, $\mathbf{r} = \begin{pmatrix} -4 \\ -6 \end{pmatrix}$ and $\mathbf{s} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$.

a. Write down two pairs of parallel vectors. [2 marks]
b. Write $\mathbf{q}$ and $\mathbf{r}$ as multiples of $\mathbf{p}$. [2 marks]
c. The vector $\begin{pmatrix} k \\ 10 \end{pmatrix}$ is parallel to $\mathbf{s}$. Find k. [2 marks]

Question 5

Question 5 – Vector journeys in a triangle *(Section A · Skills practice)*

In triangle OAB, and . M is the midpoint of OA and N is the midpoint of OB.

a. Write in terms of $\mathbf{a}$ and $\mathbf{b}$. [1 mark]

b. Write and . [2 marks]

c. Show that . What does this tell you about MN and AB? [3 marks]

Question 6

Question 6 – Position vectors and ratios *(Section A · Skills practice)*

A has position vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and B has position vector $\begin{pmatrix} 8 \\ 10 \end{pmatrix}$ relative to the origin O.

a. Find . [1 mark]

b. P lies on AB with $AP : PB = 1 : 2$. Find the position vector of P. [3 marks]
c. Find the position vector of the midpoint of AB. [2 marks]

Question 7

Question 7 – The drone delivery *(Section B · Real-life practice)*

A drone leaves a warehouse W and flies to three delivery points. Its movements are given as column vectors in kilometres: the top number is distance east and the bottom number is distance north.

Leg	Movement
W to X	$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$
X to Y	$\begin{pmatrix} -1 \\ 4 \end{pmatrix}$
Y to Z	$\begin{pmatrix} 5 \\ -3 \end{pmatrix}$

a. Find the vector . [2 marks]

b. How far is Z from the warehouse? Give your answer to 1 decimal place. [2 marks]

c. The drone flies back from Z to W in a straight line. Write down the vector for the return flight. [1 mark]

d. The drone travels at 30 km/h. How many minutes does the return flight take? [2 marks]

Question 8

Question 8 – The hiking route *(Section B · Real-life practice)*

A group of hikers walks from a campsite C to a lake L and then to a hut H. Using east and north as the directions, km and km.

a. Work out . [2 marks]

b. Work out the distance CL, the distance LH and the total distance walked. Give answers to 2 decimal places. [3 marks]

c. How much shorter is the direct route from C to H than the route through the lake? [3 marks]

Question 9

Question 9 – The treasure map *(Section B · Real-life practice)*

A treasure map says: "Start at the old oak O. The chest is at T where

paces. The key is at K where [redacted] paces."

a. Find [redacted]. [2 marks]

b. How many paces is it from the key to the chest? Give your answer to the nearest pace. [2 marks]
c. A second chest is at the midpoint of KT. Find its position vector from O. [2 marks]
d. Mira walks directly from O to T. Show that she walks 13 paces. [1 mark]

Question 10

Question 10 – A ferry crossing *(Section B · Real-life practice)*

A ferry sails across a river. Without a current it would travel $\begin{pmatrix}0\\6\end{pmatrix}$ km (east, north) in one hour. The current pushes it by $\begin{pmatrix}3\\0\end{pmatrix}$ km in that hour.

a. Work out the actual displacement of the ferry after one hour. [2 marks]
b. Find the ferry\'s actual speed in km/h to 1 decimal place. [2 marks]
c. The ferry starts at a landing stage S. How far east of the landing stage on the opposite bank does it arrive after crossing 6 km north? [2 marks]
d. The captain wants to arrive directly opposite S. What vector must she aim for so that the current cancels the sideways motion? [2 marks]

Question 11

Question 11 – A robot on a grid *(Section B · Real-life practice)*

A robot moves on a square grid. One step is $\mathbf{x} = \begin{pmatrix}2\\1\end{pmatrix}$ and another step is $\mathbf{y} = \begin{pmatrix}-1\\3\end{pmatrix}$. The robot starts at the origin.

a. Where is the robot after doing step $\mathbf{x}$ three times and step $\mathbf{y}$ twice? [2 marks]
b. Find the position after $4\mathbf{x} - \mathbf{y}$. [2 marks]
c. The robot must reach the point $(7, 7)$. It does step $\mathbf{x}$ m times and step $\mathbf{y}$ n times. Write two equations and solve them to find m and n. [4 marks]

Question 12

Question 12 – Wind and an aeroplane *(Section B · Real-life practice)*

A small plane flies on a heading that would take it $\begin{pmatrix}120\\50\end{pmatrix}$ km in one hour. A wind blows

it by $\begin{pmatrix} -30 \\ 10 \end{pmatrix}$ km per hour. The plane flies for 3 hours.

a. Find the resultant velocity of the plane over the ground. [2 marks]
b. Find the position of the plane after 3 hours relative to its starting point. [2 marks]
c. How far from the start is it? Give your answer to the nearest km. [2 marks]
d. The destination is at $\begin{pmatrix} 270 \\ 180 \end{pmatrix}$ km. Is the plane on course after 3 hours? Explain. [2 marks]

Question 13

Question 13 – A football pass *(Section B · Real-life practice)*

On a pitch plan, a midfielder M is at $(10, 20)$ metres. A striker S is at $(34, 52)$ and a winger W is at $(50, 20)$.

a. Write down the vectors and . [2 marks]

b. How long is each pass, to 1 decimal place? [3 marks]
c. A ball travels at 16 m/s. How long does the pass M to S take? [2 marks]
d. Is it shorter to pass directly from M to W, or via S? Show your working. [2 marks]

Question 14

Question 14 – Orienteering *(Section B · Real-life practice)*

In an orienteering event, runners visit three checkpoints. Relative to the start S, checkpoint A is at $\begin{pmatrix} 300 \\ 400 \end{pmatrix}$ m, checkpoint B is at $\begin{pmatrix} 900 \\ 400 \end{pmatrix}$ m and checkpoint C is at $\begin{pmatrix} 900 \\ 1000 \end{pmatrix}$ m (east, north).

a. Write down the vectors and . [2 marks]

b. Show that triangle ABC has a right angle at B. [2 marks]
c. Work out the length of the course $S \to A \to B \to C$. [3 marks]
d. A runner cuts straight from A to C. How much shorter is this than going via B? Give your answer to the nearest metre. [3 marks]

Question 15

Question 15 – The parallelogram *(Section C · Problem solving)*

$OABC$ is a parallelogram. and . M is the midpoint of AB and N is the midpoint of BC.

a. Write in terms of $\mathbf{a}$ and $\mathbf{c}$. [1 mark]

b. Write and . [3 marks]

c. Write and show that MN is parallel to AC. [3 marks]

d. What fraction of the length of AC is MN? [1 mark]

Question 16

Question 16 – Ratio on a line *(Section C · Problem solving)*

and . Point P divides AB so that $AP : PB = 2 : 3$. Point Q is on OB with $OQ : QB = 2 : 3$.

a. Find . [1 mark]

b. Show that . [3 marks]

c. Find and . [3 marks]

d. What do you notice about and ? Explain what this means for the lines. [2 marks]

Question 17

Question 17 – Showing three points are in a line *(Section C · Problem solving)*

and . , and .

a. Find and . [3 marks]

b. Show that P, Q and R lie on a straight line. [3 marks]

c. What is special about the position of P on the line QR? [2 marks]

Question 18

Question 18 – Search and rescue *(Section C · Problem solving)*

A coastguard station C is at the origin. A boat B is at $\begin{pmatrix} 12 \\ 16 \end{pmatrix}$ km and a lifeboat L is at

$\begin{pmatrix} -6 \\ 8 \end{pmatrix}$ km. A helicopter H flies from C to the midpoint of BL.

a. Find . [2 marks]

b. Find the position vector of the midpoint M of BL. [2 marks]

c. How far does the helicopter fly from C to M? Give your answer to 1 decimal place. [2 marks]

d. The helicopter flies at 150 km/h. Work out the time taken in minutes. [2 marks]

e. A second vessel is at V with . Find the coordinates of V and explain where M is on CV. [2 marks]

Question 19

Question 19 – Diagonals of a trapezium *(Section C · Problem solving)*

$OABC$ is a trapezium with OA parallel to CB and $CB = 2OA$. and . D is the midpoint of AB.

a. Write and in terms of $\mathbf{a}$ and $\mathbf{c}$. [3 marks]

b. Find . [2 marks]

c. E is on OB with $OE : EB = 1 : 2$. Find and show that D, E and C are not in a straight line unless a certain relation holds. [3 marks]

Question 20

Question 20 – Challenge: the midpoint theorem and the centroid *(Section D · Challenge)*

In triangle OAB, and . M is the midpoint of AB. G is the point on OM such that $OG : GM = 2 : 1$.

a. Find and . [3 marks]

b. N is the midpoint of OA. Show that B, G and N lie on a straight line and find the ratio $BG : GN$. [5 marks]

c. Explain why this shows that the three medians of a triangle meet at one point. [2 marks]