Answers - Vectors

Year: Year 11

Answer to Question 1

a. Add the top numbers and the bottom numbers: $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

b. Subtract: $\begin{pmatrix} 6 \\ -6 \end{pmatrix}$.

c. Multiply both numbers by 3: $\begin{pmatrix} 9 \\ 9 \end{pmatrix}$.

d. $2\mathbf{a} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$ and $3\mathbf{b} = \begin{pmatrix} -6 \\ 15 \end{pmatrix}$, so $2\mathbf{a} + 3\mathbf{b} = \begin{pmatrix} 2 \\ 13 \end{pmatrix}$.

e. $2\mathbf{c} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$, so $\mathbf{a} - 2\mathbf{c} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} - \begin{pmatrix} 6 \\ 6 \end{pmatrix} = \begin{pmatrix} -2 \\ -7 \end{pmatrix}$. Multiply $\mathbf{c}$ by 2 first, then subtract each component.

Answer to Question 2

a. $\sqrt{9+16} = \sqrt{25} =$ **5** Square each component, add, then take the square root. This is Pythagoras' theorem on the arrow of the vector.

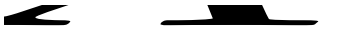
b. $\sqrt{25+144} = \sqrt{169} =$ **13** The components 5 and 12 form a 5-12-13 right-angled triangle, so the magnitude is 13.

c. $\sqrt{64+36} = \sqrt{100} =$ **10** Squaring removes the negative sign: $(-8)^2 = 64$.

d. $\sqrt{4+25} = \sqrt{29} =$ **5.39** 29 is not a square number, so round $\sqrt{29}$ to 2 decimal places.

e. $\sqrt{16+49} = \sqrt{65} =$ **8.06** Both components are negative, but the magnitude (a length) is always positive.

Answer to Question 3

a. End minus start:  .

b.  and  .

c. $\begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} -7 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. The journey $A \to B \to C \to A$ returns to the start, so the total is the **zero vector**.

d. $\sqrt{16+9} =$ **5 units**. The length of AB is the magnitude of $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$, using Pythagoras on its components 4 and 3.

Answer to Question 4

a. Parallel vectors are scalar multiples of each other. $\mathbf{q} = 3\mathbf{p}$ and $\mathbf{r} = -2\mathbf{p}$, so $\mathbf{p}, \mathbf{q}$ are parallel, $\mathbf{p}, \mathbf{r}$ are parallel and $\mathbf{q}, \mathbf{r}$ are parallel. $\mathbf{s}$ is not parallel to any of them.
b. $\mathbf{q} = 3\mathbf{p}$ because $3 \times 2 = 6$ and $3 \times 3 = 9$. $\mathbf{r} = -2\mathbf{p}$ because $-2 \times 2 = -4$ and $-2 \times 3 = -6$.
c. $5 \times 2 = 10$, so the multiplier is 2 and $k = 4 \times 2 =$ **8**.

Answer to Question 5

a. $\mathbf{b} - \mathbf{a}$. To go from A to B we go back along $\mathbf{a}$ to O, then along $\mathbf{b}$.
b. and . A midpoint is halfway along, so we take half of each vector.
c. . So MN is **parallel** to AB and **half** its length.

Answer to Question 6

a. . Subtract the position vector of the start from the position vector of the end.
b. $AP = \frac{1}{3}AB$, so . Then $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$. $AP : PB = 1 : 2$ means P is one third of the way from A to B. Check:
.
c. Midpoint $= \frac{1}{2}(\mathbf{a} + \mathbf{b}) = \frac{1}{2}\begin{pmatrix} 10 \\ 11 \end{pmatrix} = \begin{pmatrix} 5 \\ 5.5 \end{pmatrix}$. The midpoint is the mean of the two position vectors; add them and halve each component.

Answer to Question 7

a. $\begin{pmatrix}7\\3\end{pmatrix}$. The total displacement is the sum of the three movement vectors.

b. $\sqrt{7^2+3^2}=\sqrt{58}=$ **7.6 km**. The distance is the magnitude of , using Pythagoras.

c. The return vector is the negative: $\begin{pmatrix}-7\\-3\end{pmatrix}$. Going back reverses the direction, so both components change sign.

d. Time $=\dfrac{7.62}{30}=0.254$ hours $=0.254\times 60=$ **15.2 minutes** (about 15 minutes). Time = distance ÷ speed, then multiply by 60 to change hours into minutes.

Answer to Question 8

a. $\begin{pmatrix}6\\-3\end{pmatrix}$. The total displacement is the sum of the two legs.

b. $CL=\sqrt{16+9}=5$ km. $LH=\sqrt{4+36}=\sqrt{40}=6.32$ km. Total $=5+6.32=$ **11.32 km**. Each distance is the magnitude of its vector, found with Pythagoras.

c. $CH=\sqrt{36+9}=\sqrt{45}=6.71$ km. The saving is $11.32-6.71=$ **4.61 km**. The direct route is the magnitude of ; subtract it from the total distance walked.

Answer to Question 9

a. $\begin{pmatrix}16\\-3\end{pmatrix}$. Go from K back to O (the negative of), then from O to T.

b. $\sqrt{16^2+3^2}=\sqrt{265}=16.28$, so about **16 paces**. The distance is the magnitude of .

c. Midpoint $=\frac{1}{2}\left(\begin{pmatrix}12\\5\end{pmatrix}+\begin{pmatrix}-4\\8\end{pmatrix}\right)=\frac{1}{2}\begin{pmatrix}8\\13\end{pmatrix}=\begin{pmatrix}4\\6.5\end{pmatrix}$. The position vector of a midpoint is the mean of the two position vectors.

d. $\sqrt{12^2+5^2}=\sqrt{169}=$ **13** paces. The magnitude of uses the 5-12-13 right-angled triangle.

Answer to Question 10

a. $\begin{pmatrix}0\\6\end{pmatrix}+\begin{pmatrix}3\\0\end{pmatrix}=\begin{pmatrix}3\\6\end{pmatrix}$.

b. $\sqrt{9+36}=\sqrt{45}=$ **6.7 km/h**.

c. After 1 hour the ferry has gone 6 km north and has drifted **3 km east** of the point opposite S.

d. She needs a sideways component of -3 to cancel the current $\begin{pmatrix}3\\0\end{pmatrix}$, for example $\begin{pmatrix}-3\\6\end{pmatrix}$ (and a northward part such as 6).

Answer to Question 11

a. $3\mathbf{x}+2\mathbf{y}=\begin{pmatrix}6\\3\end{pmatrix}+\begin{pmatrix}-2\\6\end{pmatrix}=\begin{pmatrix}4\\9\end{pmatrix}$.

b. $4\mathbf{x}-\mathbf{y}=\begin{pmatrix}8\\4\end{pmatrix}-\begin{pmatrix}-1\\3\end{pmatrix}=\begin{pmatrix}9\\1\end{pmatrix}$.

c. Across: $2m-n=7$. Up: $m+3n=7$. From the first, $n=2m-7$. Substitute: $m+3(2m-7)=7$, so $7m-21=7$, $7m=28$ and $m=4$. Then $n=2\times4-7=$ **1**. Check: $4\mathbf{x}+\mathbf{y}=\begin{pmatrix}8\\4\end{pmatrix}+\begin{pmatrix}-1\\3\end{pmatrix}=\begin{pmatrix}7\\7\end{pmatrix}$.

Answer to Question 12

a. $\begin{pmatrix}120\\50\end{pmatrix}+\begin{pmatrix}-30\\10\end{pmatrix}=\begin{pmatrix}90\\60\end{pmatrix}$ km/h. Add the plane's own velocity and the wind velocity.

b. $3\times\begin{pmatrix}90\\60\end{pmatrix}=\begin{pmatrix}270\\180\end{pmatrix}$. Multiply the resultant velocity by the time.

c. $\sqrt{270^2+180^2}=\sqrt{105300}=$ **324 km** to the nearest km. Use Pythagoras on the components of the displacement.

d. Yes: the position $\begin{pmatrix}270\\180\end{pmatrix}$ is exactly the destination vector $\begin{pmatrix}270\\180\end{pmatrix}$, so the wind effect was already allowed for and the plane is **on course**.

Answer to Question 13

a. [redacted] and [redacted].

b. $MS = \sqrt{24^2 + 32^2} = \sqrt{1600} =$ **40.0 m**. $SW = \sqrt{16^2 + 32^2} = \sqrt{1280} =$ **35.8 m**.

c. $\frac{40}{16} =$ **2.5 seconds**. Time = distance ÷ speed.

d. $MW = \begin{pmatrix} 40 \\ 0 \end{pmatrix}$, length 40 m. Via S the ball travels $40 + 35.8 = 75.8$ m. So the **direct pass** is much shorter.

Answer to Question 14

a. [illegible] $\begin{pmatrix} 600 \\ 0 \end{pmatrix}$ and [illegible] $\begin{pmatrix} 0 \\ 600 \end{pmatrix}$.

b. [illegible] is horizontal and [illegible] is vertical, so the angle at B is 90° (check: $600 \times 0 + 0 \times 600 = 0$).

c. $SA = \sqrt{300^2 + 400^2} = 500$ m, $AB = 600$ m, $BC = 600$ m. Total $= 500 + 600 + 600 =$ **1 700 m**.

d. $AC = \sqrt{600^2 + 600^2} = \sqrt{720000} = 849$ m. Via B it is 1200 m, so the saving is $1200 - 849 =$ **351 m**.

Answer to Question 15

a. In a parallelogram [illegible], so [illegible] $\mathbf{a} + \mathbf{c}$.

b. [illegible], so [illegible]. [illegible], so [illegible].

c. [illegible]. Also [illegible]. So [illegible]: it is a multiple of [illegible], so MN is **parallel** to AC.

d. **One half.**

Answer to Question 16

a. [illegible].

b. $AP = \frac{2}{5}AB$, so [illegible]. Then [illegible]. As required.

c. .

d. , a multiple of , so PQ is **parallel** to OA and $\frac{3}{5}$ of its length (in the opposite direction).

Answer to Question 17

a. .

b. , so the two vectors are parallel. They both start at P, so P, Q and R are on the **same straight line**.

c. PR is the same length as PQ but in the opposite direction, so P is the **midpoint** of QR.

Answer to Question 18

a. $\begin{pmatrix}-18\\-8\end{pmatrix}$. Subtract the position vector of B from that of L.

b. $M = \frac{1}{2}(\begin{pmatrix}12\\16\end{pmatrix} + \begin{pmatrix}-6\\8\end{pmatrix}) = \frac{1}{2}\begin{pmatrix}6\\24\end{pmatrix} = \begin{pmatrix}3\\12\end{pmatrix}$. The midpoint is the mean of the two position vectors.

c. $CM = \sqrt{9+144} = \sqrt{153} =$ **12.4 km**. The distance is the magnitude of .

d. $\frac{12.37}{150} \times 60 =$ **4.9 minutes**. Time = distance ÷ speed, then multiply by 60.

e. $V = 2\begin{pmatrix}3\\12\end{pmatrix} = (6, 24)$. M is the **midpoint** of CV. Doubling a vector keeps its direction and doubles its length.

Answer to Question 19

a. .

b. .

c. . and



. These are parallel only if $\frac{5}{6} \div (-\frac{2}{3}) = \frac{1}{6} \div \frac{2}{3}$, which is false ($-\frac{5}{4} \neq \frac{1}{4}$). So D, E, C are **not** collinear.

Answer to Question 20

a.  .

b.  . Then

because $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$ and $\frac{2}{3} \times (-1) = -\frac{2}{3}$. So BG is a multiple of BN and they share B, so B, G, N are collinear. $BG : GN = 2 : 1$.

c. The median from B also passes through G at two thirds of its length. By symmetry the median from A does too, so all three medians meet at G, the centroid.