# Answers - Transformations

Year: Year 11

## Answer to Question 1

a. Change the sign of each $x$-coordinate: $(-1, 1)$, $(-4, 1)$, $(-1, 3)$.
b. Swap $x$ and $y$: $(1, 1)$, $(1, 4)$, $(3, 1)$.
c. The mirror is 5 units from the $y$-axis; the image of $x$ is $10 - x$: $(9, 1)$, $(6, 1)$, $(9, 3)$.
d. Reflection in $y = x$ leaves $A(1, 1)$ unchanged because $A$ lies on the line.

## Answer to Question 2

a. Subtract 3 from each $x$ and add 4 to each $y$: $(-2, 5)$, $(1, 5)$, $(-2, 7)$. The top number of the vector moves each point across and the bottom number moves it up or down.
b. The opposite vector: $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$. The reverse translation moves each point back by the same amount in the opposite direction.
c. Vector $= \begin{pmatrix} 6-1 \\ -2-1 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$. $B \to (9, -2)$, $C \to (6, 0)$. The vector is the change from $A$ to its image; apply the same change to $B$ and $C$.

## Answer to Question 3

a. $(x, y) \to (y, -x)$: $(1, -1)$, $(1, -4)$, $(3, -1)$. A $90^\circ$ clockwise turn about the origin sends $(x, y)$ to $(y, -x)$. Check: $B(4, 1)$ goes to $(1, -4)$, still 4.12 units from the origin.
b. $(x, y) \to (-y, x)$: $(-1, 1)$, $(-1, 4)$, $(-3, 1)$. A $90^\circ$ anticlockwise turn sends $(x, y)$ to $(-y, x)$. The triangle moves into the second quadrant.
c. $(x, y) \to (-x, -y)$: $(-1, -1)$, $(-4, -1)$, $(-1, -3)$. A half turn changes the sign of both coordinates, so the triangle ends up in the third quadrant.

## Answer to Question 4

a.  . Subtract the coordinates of the centre from those of $P$.

b. $(x, y) \to (-y, x)$ gives $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Turning a vector $90^\circ$ anticlockwise sends $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} -y \\ x \end{pmatrix}$.

c. Add to the centre: $(2+1, 2+2) = (3, 4)$. The rotated vector starts at the centre, so we add it to the coordinates of $C$.
d. $CP = \sqrt{4+1} = \sqrt{5}$. $CP' = \sqrt{1+4} = \sqrt{5}$. ✓ A rotation keeps every point the same distance from the centre.

## Answer to Question 5

a. Multiply every coordinate by 2: $(2, 2)$, $(8, 2)$, $(2, 6)$.
b. Vector from the centre to each point multiplied by 3. $A$ is at the centre so it stays at $(1, 1)$. $B$: $(1, 1) + 3(3, 0) = (10, 1)$. $C$: $(1, 1) + 3(0, 2) = (1, 7)$.
c. Area of $T = \frac{1}{2} \times 3 \times 2 = 3$. The image area is $3 \times 3^2 =$ **27**.

## Answer to Question 6

a. A **reflection** in the $y$-axis ($x = 0$). Each $x$-coordinate changes sign and each $y$-coordinate stays the same.
b. A **translation** by the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Every point moves 2 right and 3 up.
c. A **rotation** of $180^\circ$ about the origin (centre $(0, 0)$). Both coordinates change sign, which is a half turn about the origin.

## Answer to Question 7

a. $(x, y) \to (-y, x)$: $(0, 6)$.
b. $(-6, 0)$.
c. $270^\circ$ anticlockwise is $90^\circ$ clockwise: $(x, y) \to (y, -x)$: $(0, -6)$.
d. Distance from the origin is always 6, so the tip is on a circle of radius 6. One turn is $2\pi \times 6 =$ **37.7 cm**.

## Answer to Question 8

a. Multiply by 1.5: $(3, 3)$, $(9, 3)$, $(9, 7.5)$, $(3, 7.5)$. With centre the origin, every coordinate is multiplied by the scale factor.
b. Before: $4 \times 3 =$ **12 cm²**. After: $6 \times 4.5 =$ **27 cm²**. The original rectangle is $6 - 2 = 4$ cm by $5 - 2 = 3$ cm; the image is $9 - 3 = 6$ cm by $7.5 - 3 = 4.5$ cm.
c. $27 : 12 = 9 : 4$, which is $1.5^2 : 1$, the square of the scale factor. Divide both areas by 3 to simplify the ratio.

## Answer to Question 9

a. $\begin{pmatrix} 3-1 \\ 2-1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

b. Multiply by $-2$: $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$. Add to the centre: $(-3, -1)$.

c. The image is on the **opposite side** of the centre and is turned upside down, and its lengths are 2 times as long.

d. Area scale factor $= (-2)^2 = 4$, so the image area is $5 \times 4 =$ **20 cm²**.

## Answer to Question 10

a. . Clockwise: $(x, y) \to (y, -x)$ gives $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$. Add the centre: $(3, 0)$.

b. $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$, so $(0, 3)$.

c. $\frac{15}{40} \times 360 = 135°$. 15 seconds is $\frac{15}{40}$ of a full turn.

d. Circumference $= 2\pi \times 3 = 18.85$ cm in 40 s, so speed $=$ **0.47 cm/s**. Speed $=$ distance $\div$ time: $18.85 \div 40 \approx 0.47$.

## Answer to Question 11

a. The image of $x$ is $a + 2(x - a)$. With $x = 2$: $a + 2(2 - a) = 5$. The image of a point is the centre plus 2 times the vector from the centre to the point.

b. $a + 4 - 2a = 5$, so $-a = 1$ and $a = -1$. By symmetry $b = -1$. The centre is $(-1, -1)$. The same working with the $y$-coordinates, $b + 2(2 - b) = 5$, gives $b = -1$.

c. ; times 2 gives $\begin{pmatrix} 8 \\ 6 \end{pmatrix}$; $Q' = (-1 + 8, -1 + 6) = (7, 5)$. The image $Q'$ is twice as far from the centre as $Q$, so the centre is correct.

## Answer to Question 12

a. After the first: $(-1, 1)$, $(-4, 1)$, $(-1, 3)$. After the second: $(-1, -1)$, $(-4, -1)$, $(-1, -3)$.

b. A **rotation of $180°$ about the origin**.

c. Reflecting in the $x$-axis first gives $(1, -1)$, then in the $y$-axis gives $(-1, -1)$: the same. For these two perpendicular mirrors the order does **not** matter.

## Answer to Question 13

a. Right: $\begin{pmatrix}2\\0\end{pmatrix}$. Above: $\begin{pmatrix}0\\2\end{pmatrix}$. Each tile moves 2 units, the side of the tile, to the next position.
b. Translation $\begin{pmatrix}6\\4\end{pmatrix}$: $(6,4)$, $(8,4)$, $(8,6)$, $(6,6)$. 3 places right is $3 \times 2 = 6$ across and 2 places above is $2 \times 2 = 4$ up.
c. $6 \div 0.5 = 12$ and $4 \div 0.5 = 8$, so $12 \times 8 =$ **96 tiles**. Divide each length of the room by the side of a tile, then multiply.

## Answer to Question 14

a. $8 \times 2.5 =$ **20 cm**. Lengths are multiplied by the scale factor.
b. $12 \times 2.5^2 = 12 \times 6.25 =$ **75 cm²**. Areas are multiplied by the square of the scale factor.
c. $75 \div 6.25 =$ **12 cm²**. To go back from the copy to the original, divide by the area factor.

## Answer to Question 15

a. First reflection: $(5,2)$. Second: $(-5,2)$. Reflecting in $y = x$ swaps the coordinates; reflecting in the $y$-axis changes the sign of $x$.
b. $(x,y) \to (-y,x)$ turns $(2,5)$ into $(-5,2)$. So the combination is a **rotation of** $90°$ **anticlockwise about the origin**. The final point matches the rule for a $90°$ anticlockwise rotation about the origin.
c. $(3,1) \to (1,3) \to (-1,3)$ and $90°$ anticlockwise gives $(-1,3)$ ✓. The rule works for a second point, which supports the answer.

## Answer to Question 16

a. A **reflection** in the $x$-axis ($y = 0$). The $x$-coordinates stay the same and the $y$-coordinates change sign.
b. A **rotation of** $180°$ **about the origin**, or an **enlargement of scale factor** $-1$ with centre the origin. Both coordinates change sign, so either description gives the same image.
c. $V \to U$: $(x,y) \to (-x,y)$, a **reflection** in the $y$-axis ($x = 0$). The $y$-coordinates stay the same and the $x$-coordinates change sign.

## Answer to Question 17

a. $P' = (1,1) - 2(2,1) = (-3,-1)$. $Q' = (1,1) - 2(4,1) = (-7,-1)$.
$R' = (1,1) - 2(2,3) = (-3,-5)$. Find the vector from the centre to each point, multiply it by $-2$, then add it to the centre.

b. Area of $PQR = \frac{1}{2} \times 2 \times 2 = 2$. Image $= 2 \times (-2)^2 =$ **8**. The area scale factor is the square of the scale factor, which is positive.
c. A negative scale factor reverses the direction of every vector from the centre.

## Answer to Question 18

a. $20 \times 3^2 =$ **180 cm²**. Area is multiplied by the square of the scale factor, $3^2 = 9$.
b. $12 \times 3^3 =$ **324 cm³**. Volume is multiplied by the cube of the scale factor, $3^3 = 27$.
c. Length: $10 \times 0.5 =$ **5 cm**. Area: $40 \times 0.25 =$ **10 cm²**. Lengths are multiplied by 0.5 and areas by $0.5^2 = 0.25$, so the shape gets smaller.

## Answer to Question 19

a. $A'B' = 2$ and $AB = 1$, so the scale factor is **2**.
b. $AA'$: gradient $\frac{4-2}{3-2} = 2$, so $y = 2x - 2$. $BB'$: gradient $\frac{4-2}{5-3} = 1$, so $y = x - 1$.
c. Solve $2x - 2 = x - 1$: $x = 1$ and $y = 0$. The centre is $(1, 0)$. Check with $C$: $(1, 0) + 2(1, 4) = (3, 8)$ ✓.

## Answer to Question 20

a. After reflecting: $(1, -1)$, $(4, -1)$, $(1, -3)$. After translating: $(-1, 4)$, $(2, 4)$, $(-1, 2)$.
b. Yes: reflection and translation both preserve lengths and angles.
c. The combination of a reflection and a translation can be a single reflection only if the translation is perpendicular to the mirror line. Here the vector $(-2, 5)$ has a part parallel to the mirror $(-2)$, so it is a **glide reflection**, not a single reflection. The midpoint of $A$ and $A'$ is $(0, 2.5)$, not on the $x$-axis.

## Answer to Question 21

a. Reflection in $x = 3$: $(5, 1)$. Reflection in $x = 7$: $(9, 1)$.
b. A **translation** by $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$. The mirrors are 4 apart and $8 = 2 \times 4$.
c. $(2, 0)$ stays at $(2, 0)$ in the first reflection, then goes to $(0, 2)$. This is a **rotation of** $90^\circ$ **anticlockwise** about the origin. The mirrors meet at $45^\circ$ and $90^\circ = 2 \times 45^\circ$.