Test answers - Sine rule, cosine rule and area of a triangle

Year: Year 11 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. $\frac{b}{\sin 70^\circ} = \frac{12}{\sin 50^\circ}$ (1), $b = \frac{12 \sin 70^\circ}{\sin 50^\circ}$ (1) $=$ **14.72 cm** (1) We know a and its opposite angle A, so the sine rule gives b, the side opposite B. Multiply both sides by $\sin 70^\circ$.

b. $C = 180 - 50 - 70 = 60^\circ$ (1) The angles of a triangle add up to 180°.

Check (a): the side opposite the larger angle is longer, and $70^\circ > 50^\circ$ gives $b > a$. The side c is $12 \sin 60^\circ \div \sin 50^\circ \approx 13.57$ cm.

Answer to Question 2 (4 marks)

a. $a^2 = 81 + 121 - 2 \times 9 \times 11 \cos 62^\circ$ (1) $= 202 - 92.96 = 109.04$ (1), $a =$ **10.44 cm** (1) Two sides and the included angle are known, so we use the cosine rule, then take the square root.

b. $\frac{1}{2} \times 9 \times 11 \sin 62^\circ =$ **43.7 cm²** (1) Use $\frac{1}{2}bc \sin A$ with the two given sides and the angle between them.

In (a), a common error is to work out $(81 + 121 - 198) \cos 62^\circ$ by ignoring the order of operations. Keep the calculator value of a for later parts.

Answer to Question 3 (4 marks)

a. $\frac{1}{2} \times 7.5 \times 9 \sin 42^\circ =$ **22.6 cm²** (2) Two sides and the included angle are known, so we use $\frac{1}{2}ab \sin C$.

b. $c^2 = 56.25 + 81 - 135 \cos 42^\circ = 137.25 - 100.33 = 36.92$ (1), $c =$ **6.08 cm** (1) Use the cosine rule with the same two sides and angle, then take the square root.

In (b), check that the answer is sensible: the third side is shorter than $7.5 + 9 = 16.5$ cm and the angle opposite it, 42°, is the smallest angle, so it should be the shortest side, which 6.08 cm is.

Answer to Question 4 (4 marks)

a. $\cos x = \frac{36 + 64 - 121}{2 \times 6 \times 8}$ (1) $= \frac{-21}{96}$ (1), $x = 102.6^\circ$ (1) All three sides are known, so we use the rearranged cosine rule. The largest angle is opposite the longest side,

11 cm.

b. Obtuse (1) The largest angle is more than 90° because its cosine is negative.

In (b), the converse of Pythagoras gives the same result: $6^2 + 8^2 = 100$, which is less than $11^2 = 121$, so the angle is obtuse.

Answer to Question 5 (4 marks)

a. $d^2 = 225 + 400 - 600\cos 75^\circ$ (1) $= 625 - 155.3 = 469.7$ (1), $d =$ **21.7 km** (1) The two paths and the angle between them are known, so we use the cosine rule.

b. $\frac{1}{2} \times 15 \times 20 \sin 75^\circ =$ **144.9 km²** (1) Use $\frac{1}{2}ab\sin C$ with the two paths and the 75° angle.

A common error in (a) is to use Pythagoras, which only works if the angle is 90°. In (b), the area is in square kilometres because the lengths are in km.