Worksheet - Sine rule, cosine rule and area of a triangle

Year: Year 11 · 20

Show your working.

Question 1

Question 1 – Using the sine rule to find a side (Higher) *(Section A · Skills practice)*

In triangle ABC, side a is opposite angle A, side b is opposite angle B and side c is opposite angle C. The sine rule is $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

a. $A = 40^\circ$, $B = 65^\circ$, $a = 8$ cm. Find b to 2 decimal places. [3 marks]
b. Find angle C. [1 mark]
c. Find side c to 2 decimal places. [3 marks]

Question 2

Question 2 – Using the sine rule to find an angle (Higher) *(Section A · Skills practice)*

In triangle ABC, $a = 9$ cm, $b = 12$ cm and angle $A = 35^\circ$.

a. Use the sine rule to find acute angle B to 1 decimal place. [3 marks]
b. Explain why angle B could also be obtuse and find that value. [2 marks]
c. Which angle B is correct if the triangle is known to be acute? [1 mark]

Question 3

Question 3 – Using the cosine rule to find a side (Higher) *(Section A · Skills practice)*

The cosine rule is $a^2 = b^2 + c^2 - 2bc\cos A$.

a. In triangle ABC, $b = 7$ cm, $c = 9$ cm and $A = 48^\circ$. Find a to 2 decimal places. [3 marks]
b. Find angle B using the sine rule. [3 marks]
c. Check that the three angles add to 180° after finding C. [2 marks]

Question 4

Question 4 – Using the cosine rule to find an angle (Higher) *(Section A · Skills practice)*

A triangle has sides $a = 7$ cm, $b = 8$ cm and $c = 10$ cm.

a. Write the cosine rule in the form for finding $\cos C$. [1 mark]
b. Find the largest angle C to 1 decimal place. [3 marks]
c. Find angle A and angle B. [3 marks]

Question 5

Question 5 – Area of a triangle (Higher) *(Section A · Skills practice)*

The area of a triangle is $\frac{1}{2}ab\sin C$.

a. Two sides are 8 cm and 11 cm with the included angle 37°. Find the area to 1 decimal place. [2 marks]
b. Another triangle has sides 5 cm and 6 cm and area 12 cm². Find the included angle, acute. [3 marks]
c. Explain why $\frac{1}{2} \times 5 \times 6 \times \sin C$ cannot give an area greater than 15 cm². [1 mark]

Question 6

Question 6 – Exact values and area (Higher) *(Section A · Skills practice)*

Use exact values of sine without a calculator.

a. A triangle has sides 4 cm and 6 cm with an included angle of 30°. Find its exact area. [2 marks]
b. The included angle is now 60°. Find the exact area. [2 marks]
c. Find the area for the included angle 90° and explain which angle gives the largest area. [2 marks]

Question 7

Question 7 – Two ships (Higher) *(Section B · Real-life practice)*

Two ships leave a port P. Ship A sails 12 km on a bearing of 050°. Ship B sails 9 km on a bearing of 110°.

a. Show that angle $APB = 60^\circ$. [1 mark]
b. Find the distance AB. [3 marks]
c. Find the area of triangle APB. [2 marks]
d. Find angle PAB to 1 decimal place. [3 marks]

Question 8

Question 8 – A triangular field (Higher) *(Section B · Real-life practice)*

A field is a triangle with two sides of 120 m and 150 m and an angle of 52° between them.

a. Find the area of the field to the nearest square metre. [2 marks]
b. Find the length of the third side. [3 marks]
c. Fencing costs £12.50 per metre. Find the cost of fencing all the way round. [3 marks]

Question 9

Question 9 – Surveying with two angles (Higher) *(Section B · Real-life practice)*

Points A and B are 200 m apart on level ground. A tower T is seen from A at an angle $TAB = 62^\circ$ and from B at $TBA = 71^\circ$.

a. Find angle ATB. [1 mark]
b. Find the distance AT to 1 decimal place. [3 marks]
c. Find the distance BT. [2 marks]
d. From A the angle of elevation of the top of the tower is 12°. Find the height of the tower. [3 marks]

Question 10

Question 10 – A sail (Higher) *(Section B · Real-life practice)*

A triangular sail has sides 3.2 m, 4.1 m and 5.0 m.

a. Find the largest angle to 1 decimal place. [3 marks]
b. Find the area of the sail to 2 decimal places. [3 marks]
c. Sail cloth costs £18 per m². Find the cost of the cloth for 6 sails. [3 marks]

Question 11

Question 11 – A flight with a turn (Higher) *(Section B · Real-life practice)*

A plane flies 250 km on a bearing of 030° and then 180 km on a bearing of 100°.

a. Show that the angle between the two legs, inside the triangle, is 110°. [2 marks]
b. Find the direct distance from the start. [3 marks]
c. Find the bearing of the end point from the start to the nearest degree. [4 marks]

Question 12

Question 12 – Across a lake (Higher) *(Section B · Real-life practice)*

To find the distance across a lake, a surveyor stands at C and measures $CA = 85$ m, $CB = 120$ m and angle $ACB = 74^\circ$.

a. Find AB to 1 decimal place. [3 marks]
b. Find angle CAB. [3 marks]
c. Find the area of triangle ABC. [2 marks]

Question 13

Question 13 – An obtuse triangle (Higher) *(Section B · Real-life practice)*

A triangle has sides 5 cm, 7 cm and 10 cm.

a. Use the cosine rule to find the largest angle. [3 marks]
b. Explain how the sign of $\cos C$ shows that the angle is obtuse. [1 mark]
c. Find the other two angles. [4 marks]

Question 14

Question 14 – Finding an angle from the area (Higher) *(Section B · Real-life practice)*

A triangle has sides 10 cm and 8 cm and an area of 30 cm².

a. Show that $\sin C = 0.75$. [2 marks]
b. Find the two possible sizes of the included angle C. [3 marks]
c. Find the third side for each case. [4 marks]

Question 15

Question 15 – A quadrilateral (Higher) *(Section C · Problem solving)*

In quadrilateral $ABCD$: $AB = 6$ cm, $BC = 7$ cm, angle $ABC = 80°$, $CD = 5$ cm and $DA = 9$ cm.

a. Find AC to 2 decimal places. [3 marks]
b. Find angle ACD. [3 marks]
c. Find the area of the quadrilateral. [5 marks]

Question 16

Question 16 – Exact sine rule (Higher) *(Section C · Problem solving)*

In triangle ABC, $A = 45°$, $B = 60°$ and $a = 6$ cm.

a. Use the sine rule to show that $b = 3\sqrt{6}$. [4 marks]
b. Find angle C and the exact area using $\frac{1}{2}ab\sin C$ (leave in surd form, $\sin 75°$ left as it is). [3 marks]
c. Find b to 2 decimal places. [1 mark]

Question 17

Question 17 – Cosine rule with an angle of 120 degrees (Higher) *(Section C · Problem solving)*

A triangle has sides 5 cm and 8 cm with an included angle of $120°$.

a. Show that the third side squared is 129. [3 marks]
b. Find the third side. [1 mark]
c. Find the area exactly. [3 marks]
d. Find the smallest angle. [3 marks]

Question 18

Question 18 – Show that an angle is 120 degrees (Higher) *(Section C · Problem solving)*

A triangle has sides 7 cm, 8 cm and 13 cm.

a. Show that the largest angle is exactly 120°. [4 marks]
b. Find the exact area. [3 marks]
c. Find the height from the 13 cm side to the opposite vertex. [2 marks]

Question 19

Question 19 – Two lighthouses (Higher) *(Section C · Problem solving)*

Lighthouses A and B are 10 km apart with B due east of A. A ship S is on a bearing of 030° from A and a bearing of 310° from B.

a. Show that angle $SAB = 60^\circ$ and angle $SBA = 40^\circ$. [3 marks]
b. Find angle ASB. [1 mark]
c. Find the distance AS. [3 marks]
d. Find how far the ship is north of the line AB. [2 marks]

Question 20

Question 20 – Challenge: why the area formula works *(Section D · Challenge)*

In triangle ABC, CD is the perpendicular from C to AB.

a. Write CD in terms of a and $\sin B$. [2 marks]
b. Hence show that the area of the triangle is $\frac{1}{2}ac\sin B$. [2 marks]
c. The area can also be written as $\frac{1}{2}bc\sin A$. Equate the two expressions and show that $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$. [4 marks]