Worksheet - Similarity and congruence

Year: Year 11 · 20

Show your working.

Question 1

Question 1 – Congruent triangles *(Section A · Skills practice)*

Two triangles are congruent if one of these conditions holds: SSS, SAS, ASA (or AAS) or RHS. Decide which condition, if any, proves each pair congruent.

a. Triangle 1: sides 5 cm, 7 cm, 9 cm. Triangle 2: sides 9 cm, 5 cm, 7 cm. [2 marks]
b. Triangle 1: sides 6 cm and 8 cm with an included angle of 50°. Triangle 2: sides 6 cm and 8 cm with an included angle of 50°. [2 marks]
c. Triangle 1 has angles 40° and 60° with 7 cm between them. Triangle 2 has angles 40° and 60° and a side of 7 cm between them. [2 marks]
d. Two right-angled triangles each have hypotenuse 13 cm and one side of 5 cm. [2 marks]
e. Triangle 1 has angles 30°, 60°, 90°. Triangle 2 has angles 30°, 60°, 90° (no sides given). [2 marks]

Question 2

Question 2 – Scale factors of similar shapes *(Section A · Skills practice)*

A rectangle is 4 cm by 6 cm. A similar rectangle is 10 cm wide.

a. Find the scale factor of the enlargement. [2 marks]
b. Find the length of the larger rectangle. [2 marks]
c. Find the perimeter of each rectangle. [2 marks]
d. What is the ratio of the perimeters? Compare with the scale factor. [2 marks]

Question 3

Question 3 – A missing side *(Section A · Skills practice)*

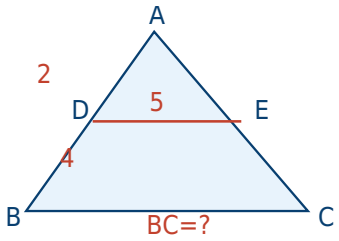
Triangle PQR has sides 6 cm, 8 cm and 10 cm. Triangle XYZ is similar with shortest side 9 cm.

a. Find the scale factor. [1 mark]
b. Find the other two sides of XYZ. [3 marks]
c. Show that XYZ is right-angled. [2 marks]

Question 4

Question 4 – A line parallel to the base *(Section A · Skills practice)*

In triangle ABC, DE is parallel to BC with D on AB and E on AC. $AD = 4$ cm, $DB = 2$ cm and $DE = 5$ cm.



a. Explain why triangles ADE and ABC are similar. [2 marks]
b. Find the scale factor from ADE to ABC. [2 marks]
c. Find BC. [2 marks]

Question 5

Question 5 – Lengths, areas and volumes *(Section A · Skills practice)*

Two similar solids have corresponding lengths in the ratio $2 : 3$.

a. Write the ratio of their surface areas. [2 marks]
b. Write the ratio of their volumes. [2 marks]
c. The smaller has surface area 36 cm². Find the surface area of the larger. [2 marks]
d. The smaller has volume 40 cm³. Find the volume of the larger. [2 marks]

Question 6

Question 6 – A proof with congruent triangles *(Section A · Skills practice)*

Triangle ABC is isosceles with $AB = AC$. M is the midpoint of BC.

a. Prove that triangles ABM and ACM are congruent. [3 marks]
b. Hence show that angle ABM = angle ACM. [1 mark]
c. Show that AM is perpendicular to BC. [3 marks]

Question 7

Question 7 – A model car *(Section B · Real-life practice)*

A model car is built to a scale of $1 : 24$. The real car is 4.32 m long.

a. Find the length of the model in centimetres. [2 marks]
b. The window on the model has area 25 cm². Find the area of the real window in m². [3 marks]
c. The real car has a boot of volume 400 litres. Find the volume of the model boot in cm³. [3 marks]

Question 8

Question 8 – Enlarging a photograph *(Section B · Real-life practice)*

A photograph measures 15 cm by 10 cm. It is enlarged so that the width becomes 24 cm.

a. Find the scale factor. [1 mark]
b. Find the new height. [1 mark]
c. By what factor does the area increase? [2 marks]
d. The original photo is printed with 3 g of ink. Estimate the ink needed for the enlargement. [2 marks]

Question 9

Question 9 – Shadows *(Section B · Real-life practice)*

A person 1.7 m tall casts a shadow 2.4 m long. At the same time a tree casts a shadow 9.6 m long.

a. Explain why the triangles are similar. [2 marks]
b. Find the height of the tree. [3 marks]
c. A flagpole is 12.5 m tall. Find the length of its shadow at the same time. [2 marks]

Question 10

Question 10 – A statue and its model (Higher) *(Section B · Real-life practice)*

A bronze statue is 3 m tall with a mass of 500 kg and a surface area of 12 m². A model of the statue is 30 cm tall and made from the same bronze.

a. Find the scale factor from statue to model. [1 mark]
b. Find the mass of the model. [3 marks]
c. Find the surface area of the model in cm². [3 marks]
d. The statue needs paint costing £4 per m². Work out the cost to paint the statue and the model. [3 marks]

Question 11

Question 11 – Similar cans *(Section B · Real-life practice)*

Two similar cans have heights 8 cm and 12 cm. The smaller holds 250 ml and has a label of area 90 cm².

a. Find the length scale factor. [1 mark]
b. Find the volume of the larger can. [3 marks]
c. Find the area of the label on the larger can. [2 marks]
d. The larger can costs £1.40 and the smaller £0.80. Which is better value per ml?

[3 marks]

Question 12

Question 12 – Area on a map *(Section B · Real-life practice)*

On a map with scale $1 : 50\,000$ a lake has an area of 3 cm².

a. Find the length that 1 cm on the map represents in km. [1 mark]
b. Find the real area of the lake in km². [4 marks]
c. A reservoir has real area 2 km². What is its area on the map? [2 marks]

Question 13

Question 13 – Two swimming pools (Higher) *(Section B · Real-life practice)*

Two similar swimming pools have depths 1.2 m and 1.8 m. The smaller has a surface area of 30 m² and a volume of 36 m³.

a. Find the length scale factor. [1 mark]
b. Find the surface area of the larger pool. [2 marks]
c. Find the volume of the larger pool. [2 marks]
d. Tiles for the floor cost £28 per m². Find the extra cost for the larger pool. [3 marks]

Question 14

Question 14 – Pizza value *(Section B · Real-life practice)*

A pizza shop sells a 8-inch pizza for £6 and a 12-inch pizza for £11. The pizzas are circles with the given diameters and the same thickness.

a. Find the area of each pizza to 2 decimal places. [3 marks]
b. Show that the area ratio is $4 : 9$. [2 marks]
c. Which pizza is better value? Show how you know. [3 marks]

Question 15

Question 15 – Parallelogram proof *(Section C · Problem solving)*

$ABCD$ is a parallelogram with diagonal AC.

a. Prove that triangles ABC and CDA are congruent. [4 marks]
b. Hence prove that opposite angles B and D are equal. [2 marks]
c. Prove that the diagonals of a parallelogram bisect each other (use triangles AMB and CMD with M the intersection). [4 marks]

Question 16

Question 16 – Similar triangles with a parallel line *(Section C · Problem solving)*

In triangle ABC, DE is parallel to BC. $AD = 6$ cm, $DB = 4$ cm, $AE = 9$ cm and $BC = 15$ cm.

a. Find EC. [3 marks]
b. Find DE. [3 marks]
c. Find the ratio of the area of triangle ADE to the area of trapezium $DBCE$. [4 marks]

Question 17

Question 17 – Scale factors from areas and volumes (Higher) *(Section C · Problem solving)*

Two similar shapes have areas 20 cm² and 45 cm². Two similar solids have volumes 54 cm³ and 128 cm³.

a. Find the length scale factor for the shapes. [3 marks]
b. The smaller shape has a perimeter of 24 cm. Find the perimeter of the larger. [2 marks]
c. Find the length scale factor for the solids. [3 marks]
d. The smaller solid has surface area 36 cm². Find the surface area of the larger. [3 marks]

Question 18

Question 18 – Similar solids (Higher) *(Section C · Problem solving)*

Two similar solids have volumes 125 cm³ and 216 cm³. The smaller solid has surface area 75 cm².

a. Find the length scale factor. [2 marks]
b. Find the surface area of the larger solid. [3 marks]
c. Write the ratio of the volumes in its simplest form. [1 mark]

Question 19

Question 19 – Kite proof *(Section C · Problem solving)*

$ABCD$ is a kite with $AB = AD$ and $CB = CD$.

a. Prove that triangles ABC and ADC are congruent. [3 marks]
b. Prove that AC bisects angle BAD. [2 marks]
c. Prove that AC bisects angle BCD. [1 mark]
d. Prove that AC is perpendicular to BD (hint: use the point where they meet). [4 marks]

Question 20

Question 20 – Challenge: the altitude to the hypotenuse *(Section D · Challenge)*

Triangle ABC has a right angle at A with $AB = 6$ cm and $AC = 8$ cm. AD is perpendicular to BC with D on BC.

a. Find BC. [1 mark]
b. Prove that triangles ABD and CBA are similar and find BD. [5 marks]
c. Find AD and DC. [3 marks]
d. Show that $AD^2 = BD \times DC$. [2 marks]