Answers - Similarity and congruence

Year: Year 11

Answer to Question 1

a. **SSS**: all three sides are equal.
b. **SAS**: two sides and the included angle.
c. **ASA**: two angles and the side between them.
d. **RHS**: right angle, hypotenuse and one side. They are congruent.
e. Not congruent: equal angles (AAA) give similar triangles of any size, so we cannot conclude they are congruent.

Answer to Question 2

a. $\frac{10}{4} = \mathbf{2.5}$. Divide the corresponding widths: large $\div$ small.
b. $6 \times 2.5 = \mathbf{15\ cm}$. Multiply the corresponding length by the scale factor.
c. Small: $2(4+6) = \mathbf{20\ cm}$. Large: $2(10+15) = \mathbf{50\ cm}$. The perimeter is twice the sum of the length and the width.
d. $50 : 20 = 2.5 : 1$, the same as the scale factor: perimeters scale like lengths.

Answer to Question 3

a. $\frac{9}{6} = \mathbf{1.5}$. Divide the corresponding shortest sides: $9 \div 6$.
b. $8 \times 1.5 = \mathbf{12\ cm}$ and $10 \times 1.5 = \mathbf{15\ cm}$. Multiply each side of PQR by the scale factor.
c. $9^2 + 12^2 = 81 + 144 = 225 = 15^2$, so by the converse of Pythagoras it is right-angled. Similar shapes have the same angles, and PQR is a 3-4-5 type triangle, so XYZ has a right angle too.

Answer to Question 4

a. Angle A is common, and angles $ADE = ABC$ and $AED = ACB$ (corresponding angles between parallel lines). So the triangles have equal angles and are similar (AA).
b. $AB = 4 + 2 = 6$, so the scale factor is $\frac{6}{4} = \mathbf{1.5}$.
c. $BC = 5 \times 1.5 = \mathbf{7.5\ cm}$.

Answer to Question 5

a. $2^2 : 3^2 = 4 : 9$. For similar solids the area scale factor is the square of the length scale factor.
b. $2^3 : 3^3 = 8 : 27$. The volume scale factor is the cube of the length scale factor.
c. $36 \times \frac{9}{4} =$ **81 cm²**. Going from small to large we multiply by the area factor $\frac{9}{4} = 2.25$.
d. $40 \times \frac{27}{8} =$ **135 cm³**. Going from small to large we multiply by the volume factor $\frac{27}{8} = 3.375$.

Answer to Question 6

a. $AB = AC$ (given), $BM = CM$ (M is the midpoint), AM is common. So the triangles are congruent by **SSS**.
b. Corresponding angles in congruent triangles are equal, so angle $ABM =$ angle ACM (base angles of an isosceles triangle).
c. Angle $AMB =$ angle AMC (congruent), and they add to 180° on the straight line BC, so each is 90°.

Answer to Question 7

a. $432 \div 24 =$ **18 cm**. Change 4.32 m into 432 cm, then divide by 24.
b. Area scale factor $= 24^2 = 576$. $25 \times 576 = 14\,400$ cm² = **1.44 m²**.
c. Volume scale factor $= 24^3 = 13\,824$. 400 litres $= 400\,000$ cm³, and $400\,000 \div 13\,824 =$ **28.9 cm³**. Change litres into cm³ first: 1 litre is 1000 cm³. Divide because we go from real to model.

Answer to Question 8

a. $\frac{24}{15} =$ **1.6**. Divide the new width by the old width.
b. $10 \times 1.6 =$ **16 cm**. Multiply the old height by the scale factor. Check: $24 : 16 = 15 : 10$.
c. $1.6^2 =$ **2.56**. The area scale factor is the square of the length scale factor. Check: $\frac{24 \times 16}{15 \times 10} = \frac{384}{150} = 2.56$.
d. $3 \times 2.56 =$ **7.68 g**. The ink covers the area, so it scales with the area factor.

Answer to Question 9

a. The sun's rays are parallel, so the angles of elevation are equal; both objects are vertical. The triangles have equal angles.
b. Scale factor $= \frac{9.6}{2.4} = 4$. Height $= 1.7 \times 4 =$ **6.8 m**.
c. $\frac{12.5}{1.7} \times 2.4 =$ **17.6 m**.

Answer to Question 10

a. $\frac{30}{300} = \frac{1}{10}$. Change 3 m into 300 cm so that both heights are in the same units.
b. Volume factor $= \left(\frac{1}{10}\right)^3 = \frac{1}{1000}$. Mass $= 500 \div 1000 =$ **0.5 kg**. Mass depends on volume, so it scales with the cube of the length factor.
c. Area factor $= \frac{1}{100}$. $12 \div 100 = 0.12$ m² $=$ **1 200 cm²**. Area scales with the square of the length factor. 1 m² is 10 000 cm².
d. Statue: $12 \times 4 =$ **£48**. Model: $0.12 \times 4 =$ **£0.48**. Multiply each area by the price per square metre.

Answer to Question 11

a. $\frac{12}{8} =$ **1.5**. Divide the corresponding heights.
b. $250 \times 1.5^3 = 250 \times 3.375 =$ **843.75 ml**. Volume scales with the cube of the length scale factor.
c. $90 \times 1.5^2 =$ **202.5 cm²**. Area scales with the square of the length scale factor.
d. Small: $\frac{80}{250} = 0.32$p per ml. Large: $\frac{140}{843.75} = 0.166$p per ml. The **larger can** is better value.

Answer to Question 12

a. $50\,000$ cm $= 0.5$ km. Every centimetre on the map is 50 000 cm in real life, and 100 000 cm is 1 km.
b. 1 cm² on the map is $0.5 \times 0.5 = 0.25$ km². So $3 \times 0.25 =$ **0.75 km²**. For areas, the scale is squared.
c. $2 \div 0.25 =$ **8 cm²**. Divide the real area by the area that 1 cm² represents.

Answer to Question 13

a. $\frac{1.8}{1.2} = 1.5$. **1.5**. Divide the corresponding depths.
b. $30 \times 1.5^2 = 67.5$ **67.5 m²**. Surface area scales with the square of the length factor, $1.5^2 = 2.25$.
c. $36 \times 1.5^3 = 121.5$ **121.5 m³**. Volume scales with the cube of the length factor, $1.5^3 = 3.375$.
d. Extra area $= 67.5 - 30 = 37.5$ m². Cost $= 37.5 \times 28 = 1050$ **£1 050**. Multiply the extra floor area by the cost per square metre.

Answer to Question 14

a. Radii 4 in and 6 in: $\pi \times 16 = 50.27$ **50.27 in²** and $\pi \times 36 = 113.10$ **113.10 in²**.
b. Ratio of diameters $8 : 12 = 2 : 3$, so the area ratio is $2^2 : 3^2 = 4 : 9$. $16 : 36 = 4 : 9$ (.)
c. Price per square inch: small $\frac{6}{50.27} = £0.119$; large $\frac{11}{113.10} = £0.097$. The **12-inch pizza** is better value.

Answer to Question 15

a. $AB = CD$ and $BC = DA$ (opposite sides of a parallelogram) and AC is common. So the triangles are congruent by **SSS**.
b. Corresponding angles are equal: angle ABC = angle CDA.
c. $AB = CD$; angle MAB = angle MCD and angle MBA = angle MDC (alternate angles, $AB \parallel DC$). So triangles AMB and CMD are congruent (ASA), giving $AM = CM$ and $BM = DM$.

Answer to Question 16

a. $\frac{AD}{DB} = \frac{AE}{EC}$, so $\frac{6}{4} = \frac{9}{EC}$ and $EC =$ **6 cm**.
b. $\frac{DE}{BC} = \frac{AD}{AB} = \frac{6}{10}$, so $DE = 0.6 \times 15 =$ **9 cm**.
c. Area scale factor from ADE to ABC is $\left(\frac{10}{6}\right)^2 = \frac{100}{36} = \frac{25}{9}$. If ADE has area 9 units then ABC has 25 and $DBCE$ has $25 - 9 = 16$. The ratio is $9 : 16$.

Answer to Question 17

a. Area ratio $\frac{45}{20} = \frac{9}{4}$. The length scale factor is $\sqrt{\frac{9}{4}} = \mathbf{1.5}$.
b. $24 \times 1.5 = \mathbf{36\ cm}$. Perimeter is a length, so it scales with the length factor.
c. Volume ratio $\frac{128}{54} = \frac{64}{27}$. The length scale factor is $\sqrt[3]{\frac{64}{27}} = \frac{4}{3}$.
d. Area factor $= \left(\frac{4}{3}\right)^2 = \frac{16}{9}$. $36 \times \frac{16}{9} = \mathbf{64\ cm^2}$. Surface area scales with the square of the length factor.

Answer to Question 18

a. $\sqrt[3]{\frac{216}{125}} = \frac{6}{5} = \mathbf{1.2}$. The volume ratio is $\frac{216}{125}$, and the length factor is its cube root: $\sqrt[3]{216} = 6$ and $\sqrt[3]{125} = 5$.
b. Area factor $= 1.2^2 = 1.44$. $75 \times 1.44 = \mathbf{108\ cm^2}$. Surface area scales with the square of the length factor.
c. $125 : 216$ (it cannot be simplified). 125 and 216 have no common factor except 1, so the ratio is already in its simplest form.

Answer to Question 19

a. $AB = AD$ (given), $CB = CD$ (given) and AC is common. Congruent by **SSS**.
b. Corresponding angles $BAC = DAC$, so AC bisects angle BAD.
c. Similarly angle $BCA =$ angle DCA.
d. Let AC meet BD at M. Triangles ABM and ADM have $AB = AD$, angle $BAM =$ angle DAM and AM common, so they are congruent (SAS). Then angle $AMB =$ angle AMD, and they add to $180°$, so each is $90°$.

Answer to Question 20

a. $\sqrt{36 + 64} = \mathbf{10\ cm}$.
b. Both triangles have a right angle and share the angle at B, so they are similar (AA). $\frac{BD}{BA} = \frac{BA}{BC}$, so $BD = \frac{6^2}{10} = \mathbf{3.6\ cm}$.
c. $DC = 10 - 3.6 = \mathbf{6.4\ cm}$. $AD = \sqrt{36 - 3.6^2} = \sqrt{23.04} = \mathbf{4.8\ cm}$.
d. $BD \times DC = 3.6 \times 6.4 = 23.04 = 4.8^2 = AD^2$. ✓