Worksheet - Rates of change and areas under graphs

Year: Year 11 · 20

Show your working.

Question 1

Question 1 – Reading a distance-time graph *(Section A · Skills practice)*

Sam cycles to a village and back. The points on his distance-time graph are joined by straight lines.

Time (hours)	0	2	2.5	4.5
Distance from home (km)	0	60	60	0

a. Work out his speed for the first part of the journey. [2 marks]
b. What happened between 2 and 2.5 hours? [1 mark]
c. Work out his speed on the way home. [2 marks]
d. Work out his average speed for the whole journey. [3 marks]

Question 2

Question 2 – Units of speed and acceleration *(Section A · Skills practice)*

Convert and calculate.

a. Change 72 km/h into m/s. [2 marks]
b. Change 15 m/s into km/h. [2 marks]
c. A car increases its speed from 0 to 24 m/s in 8 seconds. Work out its acceleration. [2 marks]
d. State the units of acceleration. [1 mark]

Question 3

Question 3 – Area under a velocity-time graph *(Section A · Skills practice)*

A train travels between two stations. Its velocity-time graph has these corner points: $(0, 0)$, $(10, 20)$, $(30, 20)$, $(40, 0)$, with time in seconds and velocity in m/s.

a. Work out the acceleration for the first 10 seconds. [2 marks]
b. Work out the distance travelled while the speed is constant. [2 marks]
c. Work out the total distance between the stations. [4 marks]
d. Work out the average speed for the journey. [2 marks]

Question 4

Question 4 – Gradient as a rate *(Section A · Skills practice)*

Water flows into a tank at a steady rate. After 5 minutes there are 150 litres in the tank. The graph of volume against time is a straight line through the origin.

a. Work out the gradient of the graph and state its units. [2 marks]
b. What does the gradient represent? [1 mark]
c. How long does it take to fill a 450-litre tank? [2 marks]
d. The flow is reduced to 20 litres per minute. How does the graph change? [1 mark]

Question 5

Question 5 – Rates in different units *(Section A · Skills practice)*

A tap delivers water at 12 litres per minute.

a. Write the rate in litres per second. [2 marks]
b. Write the rate in litres per hour. [1 mark]
c. How long does the tap take to fill a 30-litre bucket? [1 mark]
d. A hose delivers 0.5 litres per second. Which is faster? [2 marks]

Question 6

Question 6 – Estimating distance with trapezia (Higher) *(Section A · Skills practice)*

The speed of a car is recorded every 2 seconds.

Time (s)	0	2	4	6
Speed (m/s)	0	6	10	12

a. Use three trapezia to estimate the distance travelled in 6 seconds. [4 marks]
b. Is the estimate an overestimate or an underestimate if the graph curves like a "bend" (concave down)? Explain. [2 marks]
c. Work out the average speed over the 6 seconds. [1 mark]

Question 7

Question 7 – A run to the park *(Section B · Real-life practice)*

Zoe runs from home to the park, rests, then runs on to the shops. Her distance-time graph has corner points $(0, 0)$, $(1.5, 45)$, $(2, 45)$, $(3.5, 90)$ with time in hours and distance in km.

a. Work out her speed in each running section. [3 marks]
b. For how long does she rest? [1 mark]
c. Work out her average speed for the whole trip including the rest. [3 marks]

d. Work out her average speed while actually running. [3 marks]

Question 8

Question 8 – A bus journey *(Section B · Real-life practice)*

A bus leaves a station and travels along a route. Corner points of its distance-time graph: $(0, 0)$, $(0.5, 15)$, $(1, 15)$, $(2, 60)$ with time in hours and distance in km.

a. Work out the speed for each moving section. [3 marks]
b. What does the horizontal section show? [1 mark]
c. Work out the average speed for the whole journey. [2 marks]
d. The bus timetable says the journey should take 1.5 hours. How much late is the bus? [2 marks]

Question 9

Question 9 – Car acceleration *(Section B · Real-life practice)*

A sports car accelerates from rest to 100 km/h in 8 seconds.

a. Change 100 km/h into m/s to 2 decimal places. [2 marks]
b. Work out the average acceleration. [2 marks]
c. The car continues to accelerate at the same rate. How fast is it going after 12 seconds? Give your answer in km/h. [3 marks]
d. Work out the distance covered in the first 8 seconds, assuming constant acceleration. [3 marks]

Question 10

Question 10 – A lift in a tall building *(Section B · Real-life practice)*

A lift accelerates uniformly at 2 m/s² for 3 seconds, then travels at constant speed for 5 seconds and then slows uniformly to rest in 4 seconds.

a. Find the top speed. [1 mark]
b. Sketch the velocity-time graph (describe its corner points). [3 marks]
c. Work out the total distance. [4 marks]
d. Work out the deceleration in the last section. [2 marks]

Question 11

Question 11 – Filling a swimming pool *(Section B · Real-life practice)*

A pool is 6 m long, 3 m wide and 1.2 m deep. A pump delivers 25 litres per minute. 1 $\text{m}^3 = 1\,000$ litres.

a. Work out the volume of the pool in litres. [3 marks]
b. How long does it take to fill the pool? Give your answer in hours and minutes. [3

marks]
c. A second pump delivers 50 litres per minute. How long do the two pumps take together? [3 marks]

Question 12

Question 12 – Cycling speeds *(Section B · Real-life practice)*

A cyclist travels at 24 km/h.

a. Write the speed in m/s to 2 decimal places. [2 marks]
b. How long does she take to travel 500 m? [2 marks]
c. How far does she travel in 45 minutes? [2 marks]
d. A runner travels 5 m/s. Who is faster and by how much in km/h? [3 marks]

Question 13

Question 13 – Population change *(Section B · Real-life practice)*

The population of a town was 50 000 in 2010 and 56 000 in 2020. Assume the graph of population against year is a straight line.

a. Work out the gradient and state its meaning. [3 marks]
b. Predict the population in 2030. [2 marks]
c. The mayor says the town grows by 1 000 people per year. Is this correct? [1 mark]
d. In which year does the population reach 60 000? [2 marks]

Question 14

Question 14 – Cooling coffee *(Section B · Real-life practice)*

A cup of coffee at 80°C cools to 50°C after 6 minutes.

a. Work out the average rate of change of temperature. [2 marks]
b. Explain the sign of your answer. [1 mark]
c. If it cooled at this steady rate, when would it reach 20°C? [2 marks]
d. In reality the cooling slows down. What would the graph look like? [2 marks]

Question 15

Question 15 – A three-stage journey *(Section C · Problem solving)*

A car accelerates from rest to 12 m/s in 4 seconds, travels at 12 m/s for 6 seconds and then slows to rest in 3 seconds.

a. Work out the acceleration in the first stage. [2 marks]
b. Work out the deceleration in the last stage. [2 marks]
c. Work out the total distance. [4 marks]

d. Work out the average speed for the whole journey. [2 marks]

Question 16

Question 16 – Overtaking *(Section C · Problem solving)*

Car A leaves a town at 50 km/h. Car B leaves the same town half an hour later at 75 km/h on the same road.

a. Write the distance of each car from the town, s km, after t hours since A left. [3 marks]
b. Find when B catches up with A. [3 marks]
c. How far from the town is this? [1 mark]
d. Describe how the two lines would look on a distance-time graph. [2 marks]

Question 17

Question 17 – A curved velocity-time graph (Higher) *(Section C · Problem solving)*

The velocity of a cyclist is recorded every 2 seconds.

Time (s)	0	2	4	6	8
Velocity (m/s)	0	5	8	10	11

a. A tangent to the curve at $t = 4$ passes through $(2, 3)$ and $(6, 11)$. Find the acceleration at $t = 4$. [3 marks]
b. Use four trapezia to estimate the distance travelled. [4 marks]
c. Is it an overestimate or underestimate? Explain. [2 marks]

Question 18

Question 18 – A leaking tank *(Section C · Problem solving)*

A tap fills a 400-litre tank at 15 litres per minute but a leak loses 3 litres per minute.

a. Work out the net rate of filling. [1 mark]
b. How long does it take to fill the tank? [2 marks]
c. How many litres are lost to the leak in this time? [2 marks]
d. The leak is mended after 10 minutes. How long does it take to fill from that point? [3 marks]

Question 19

Question 19 – A cheetah *(Section C · Problem solving)*

A cheetah runs 100 m in 4.2 seconds.

a. Work out its average speed in m/s. [2 marks]

b. Convert the answer to km/h. [2 marks]
c. A car travels at 85 km/h. Which is faster? [1 mark]
d. The cheetah can keep this speed for 20 seconds. How far does it go? [2 marks]

Question 20

Question 20 – Challenge: how good is the estimate? *(Section D · Challenge)*

The velocity of a particle is $v = t(8 - t)$ m/s for $0 \le t \le 8$. The true distance travelled is $\frac{256}{3}$ m.

a. Complete a table for $t = 0, 2, 4, 6, 8$. [2 marks]
b. Use four trapezia to estimate the distance. [4 marks]
c. Find the percentage error in your estimate. [3 marks]
d. How could the estimate be improved? [1 mark]