Answers - Rates of change and areas under graphs

Year: Year 11

Answer to Question 1

a. Speed $= \frac{60}{2} =$ **30 km/h**.
b. The graph is horizontal, so he was **stationary** (resting) for half an hour.
c. He travels 60 km in $4.5 - 2.5 = 2$ hours: **30 km/h**.
d. Total distance $= 60 + 60 = 120$ km, total time $= 4.5$ h. Average speed $= \frac{120}{4.5} =$ **26.7 km/h**.

Answer to Question 2

a. $72 \times 1000 \div 3600 =$ **20 m/s**. Multiply by 1000 to change kilometres into metres and divide by 3600 to change hours into seconds. Dividing by 3.6 does both at once.
b. $15 \times 3600 \div 1000 =$ **54 km/h**. Do the reverse: multiply by 3.6. So 15 m/s is 54 km/h.
c. Acceleration $= \frac{24 - 0}{8} =$ **3 m/s²**. Acceleration is the change in velocity divided by the time taken.
d. Metres per second per second, **m/s²**. The velocity changes by a number of metres per second in each second.

Answer to Question 3

a. $\frac{20 - 0}{10} =$ **2 m/s²**.
b. $20 \times (30 - 10) =$ **400 m**.
c. Triangle $\frac{1}{2} \times 10 \times 20 = 100$; rectangle 400; triangle $\frac{1}{2} \times 10 \times 20 = 100$. Total = **600 m**. (Check with a trapezium: parallel sides 40 s and 20 s, height 20: $\frac{1}{2}(40 + 20) \times 20 = 600$.)
d. $\frac{600}{40} =$ **15 m/s**.

Answer to Question 4

a. Gradient $= \frac{150}{5} =$ **30 litres per minute**. The gradient is the change in volume divided by the change in time.
b. The gradient is the **rate of flow** of the water.
c. $450 \div 30 =$ **15 minutes**. Divide the volume by the rate of flow.
d. The line is **less steep** with gradient 20. A smaller rate gives a smaller gradient, so the line is less steep.

Answer to Question 5

a. $12 \div 60 =$ **0.2 litres/s**. There are 60 seconds in a minute, so divide by 60.
b. $12 \times 60 =$ **720 litres/h**. There are 60 minutes in an hour, so multiply by 60.
c. $30 \div 12 =$ **2.5 minutes**. Time = amount ÷ rate.
d. The hose: $0.5 \times 60 = 30$ litres per minute, which is **faster** than 12. Change both rates into the same units before comparing.

Answer to Question 6

a. Area $= \frac{1}{2} \times 2 \times (0 + 6) + \frac{1}{2} \times 2 \times (6 + 10) + \frac{1}{2} \times 2 \times (10 + 12) = 6 + 16 + 22 =$ **44 m**.
b. The curve bends upwards less and less, so the straight tops of the trapezia lie **below** the curve: the estimate is an **underestimate**.
c. $\frac{44}{6} =$ **7.3 m/s**. Average speed = total distance ÷ total time.

Answer to Question 7

a. First: $\frac{45}{1.5} =$ **30 km/h**. Second: $\frac{90 - 45}{3.5 - 2} = \frac{45}{1.5} =$ **30 km/h**. Speed is the gradient of each sloping part: distance divided by time.
b. $2 - 1.5 =$ **0.5 hours**. The horizontal part shows no change in distance.
c. $\frac{90}{3.5} =$ **25.7 km/h**. Divide the total distance by the total time, including the rest.
d. Running time $= 1.5 + 1.5 = 3$ h. $\frac{90}{3} =$ **30 km/h**. Leave out the half-hour rest from the time.

Answer to Question 8

a. First: $\frac{15}{0.5} =$ **30 km/h**. Last: $\frac{60 - 15}{1} =$ **45 km/h**.
b. The bus is **stopped** for half an hour (a break).

c. $\frac{60}{2} =$ **30 km/h**. Divide the total distance by the total time of 2 hours, including the stop.
d. It took 2 hours, so it is **30 minutes late**.

Answer to Question 9

a. $100 \div 3.6 =$ **27.78 m/s**. Divide by 3.6 to change km/h into m/s.
b. $\frac{27.78}{8} =$ **3.47 m/s²**. Acceleration is the change in speed divided by the time.
c. $3.47 \times 12 = 41.7$ m/s $= 41.7 \times 3.6 =$ **150 km/h**. Multiply the acceleration by the time, then multiply by 3.6 to change back to km/h.
d. Distance $= \frac{1}{2} \times 8 \times 27.78 =$ **111 m**. The distance is the area under the velocity-time graph, which is a triangle here.

Answer to Question 10

a. $2 \times 3 =$ **6 m/s**. Starting from rest, the speed increases by 2 m/s each second for 3 seconds.
b. Corner points: $(0, 0)$, $(3, 6)$, $(8, 6)$, $(12, 0)$.
c. Acceleration phase $\frac{1}{2} \times 3 \times 6 = 9$ m. Constant: $5 \times 6 = 30$ m. Deceleration phase $\frac{1}{2} \times 4 \times 6 = 12$ m. Total $= 9 + 30 + 12 =$ **51 m**. The distance is the area under the velocity-time graph: two triangles and a rectangle.
d. $\frac{6}{4} =$ **1.5 m/s²** (deceleration). The speed falls from 6 m/s to 0 in 4 seconds.

Answer to Question 11

a. $6 \times 3 \times 1.2 = 21.6$ m³ $=$ **21 600 litres**. The pool is a cuboid, so multiply the three dimensions. Then multiply by 1000 to change m³ into litres.
b. $21\,600 \div 25 = 864$ minutes $= 14.4$ hours $=$ **14 hours 24 minutes**. Divide the volume by the rate, then change minutes into hours: $0.4 \times 60 = 24$ minutes.
c. Together: 75 litres per minute. $21\,600 \div 75 = 288$ minutes $=$ **4 hours 48 minutes**. Two pumps working together add their rates: $25 + 50 = 75$ litres per minute.

Answer to Question 12

a. $24 \div 3.6 =$ **6.67 m/s**. Divide by 3.6 to change km/h into m/s.
b. $500 \div 6.67 =$ **75 seconds**. Time $=$ distance $\div$ speed, using metres and m/s.
c. $24 \times \frac{45}{60} =$ **18 km**. 45 minutes is $\frac{3}{4}$ of an hour.

d. Runner: $5 \times 3.6 = 18$ km/h. The cyclist is faster by $24 - 18 =$ **6 km/h**. Change the runner's speed into km/h by multiplying by 3.6.

Answer to Question 13

a. $\dfrac{56000 - 50000}{2020 - 2010} =$ **600 people per year**. The gradient is the change in population divided by the change in years.
b. $56\,000 + 10 \times 600 =$ **62 000**. Add 10 years of growth at 600 people per year.
c. No, the gradient shows **600** per year. The rate of 1000 per year is bigger than the gradient.
d. $60\,000 - 56\,000 = 4\,000$, and $4000 \div 600 = 6.7$, so in about **2027**. Divide the extra people needed by the yearly increase, then add to 2020.

Answer to Question 14

a. $\dfrac{50 - 80}{6} = -5$**°C per minute**.
b. The negative sign shows the temperature is **falling**.
c. From 80°C, falling 5 each minute: $60 \div 5 =$ **12 minutes**.
d. It would be a **curve** that gets less steep over time (a decreasing gradient).

Answer to Question 15

a. $\dfrac{12}{4} =$ **3 m/s²**. Acceleration is the change in velocity divided by the time.
b. $\dfrac{12}{3} =$ **4 m/s²** (deceleration). The velocity falls from 12 m/s to 0 in 3 seconds.
c. $\frac{1}{2} \times 4 \times 12 = 24$; $6 \times 12 = 72$; $\frac{1}{2} \times 3 \times 12 = 18$. Total = **114 m**. The total distance is the area under the graph: two triangles and a rectangle.
d. $\dfrac{114}{13} =$ **8.77 m/s**. The total time is $4 + 6 + 3 = 13$ seconds.

Answer to Question 16

a. A: $s = 50t$. B: $s = 75(t - 0.5)$ for $t \geq 0.5$.
b. $50t = 75t - 37.5$, so $25t = 37.5$ and $t = 1.5$ hours after A left.
c. $50 \times 1.5 =$ **75 km**.
d. B's line starts at $(0.5, 0)$ and is steeper than A's line from the origin; the lines cross at $(1.5, 75)$.

Answer to Question 17

a. Gradient of the tangent $= \dfrac{11-3}{6-2} =$ **2 m/s²**.
b. Area $= \frac{1}{2} \times 2 \times [(0+5)+(5+8)+(8+10)+(10+11)] = 5+13+18+21 =$ **57 m**.
c. The graph is concave (curves downward), so the chords lie below the curve: the estimate is an **underestimate**.

Answer to Question 18

a. $15-3 =$ **12 litres per minute**.
b. $400 \div 12 =$ **33.3 minutes**.
c. $3 \times 33.3 =$ **100 litres**.
d. After 10 minutes: $10 \times 12 = 120$ L. Remaining 280 L at 15 L/min: $280 \div 15 = 18.7$ min. Total **28.7 minutes**.

Answer to Question 19

a. $100 \div 4.2 =$ **23.8 m/s**. Average speed = distance ÷ time.
b. $23.8 \times 3.6 =$ **85.7 km/h**. Multiply by 3.6 to change m/s into km/h.
c. The cheetah is slightly faster than 85 km/h. 85.7 km/h is a little more than 85 km/h.
d. $23.8 \times 20 =$ **476 m**. Distance = speed × time.

Answer to Question 20

a. $v = 0, 12, 16, 12, 0$. Substitute each value of t into $t(8-t)$: for example $2 \times 6 = 12$.
b. Area $= \frac{1}{2} \times 2 \times [(0+12)+(12+16)+(16+12)+(12+0)] = 12+28+28+12 =$ **80 m**. Each trapezium has width 2, so its area is $\frac{1}{2} \times 2 \times$ the sum of the parallel sides.
c. True $= 85.33$. Error $= \dfrac{85.33-80}{85.33} =$ **6.25%** (underestimate). Percentage error $= \dfrac{\text{error}}{\text{true value}} \times 100$.
d. Use more, narrower strips. Narrower strips follow the curve more closely, so the error is smaller.