Test answers - Pythagoras and trigonometry in right-angled triangles

Year: Year 11 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. $\sqrt{169 - 25} = \sqrt{144} =$ **12 cm** (1 method, 1 answer) Subtract the square of the known side from the square of the hypotenuse, then take the square root. The other side is 12 cm.

b. $\sin x = \dfrac{5}{13}$ (1), $x = 22.6^\circ$ (1) The smallest angle is opposite the smallest side, 5 cm. Use sine with the hypotenuse 13 cm, then $\sin^{-1}$.

Check (a): 5, 12, 13 is a Pythagorean triple, since $25 + 144 = 169$.

Answer to Question 2 (4 marks)

a. $7 \tan 38^\circ =$ **5.47 cm** (1 for the equation, 1 for the answer) The adjacent side is known and we need the opposite side, so we use tangent: opposite $=$ adjacent $\times \tan 38^\circ$.

b. $\dfrac{7}{\cos 38^\circ} =$ **8.88 cm** (1 for the equation, 1 for the answer) Use cosine: hypotenuse $=$ adjacent $\div \cos 38^\circ$. Check with Pythagoras: $\sqrt{7^2 + 5.47^2} \approx 8.88$.

In (a), the calculator must be in degree mode; in radian mode $\tan 38$ gives a meaningless value. Rounding 5.47 too early would spoil the check in (b).

Answer to Question 3 (4 marks)

a. $\tan x = \dfrac{18}{30} = 0.6$ (1), $x = 31.0^\circ$ (2) The height of the pole is opposite the angle and the 30 m distance is adjacent, so we use tangent and then $\tan^{-1}$.

b. $\sqrt{900 + 324} =$ **35.0 m** (1) The line from the person to the top is the hypotenuse, so we use Pythagoras with 30 m and 18 m.

Check (b) with trigonometry: $30 \div \cos 31.0^\circ \approx 35.0$ m. The exact value is $\sqrt{1224} \approx 34.99$ m. A common error in (a) is to use sine with 18 and 30, treating 30 m as the hypotenuse.

Answer to Question 4 (4 marks)

a. $\cos 60^\circ = \frac{1}{2}$ (1) and $\tan 45^\circ = 1$ (1) Read the exact values from the table of special angles. In a right-angled isosceles triangle the opposite and adjacent sides are equal, so $\tan 45^\circ = 1$.

b. $8 \sin 30^\circ = 8 \times \frac{1}{2} =$ **4 cm** (2) The side opposite the 30° angle uses sine: opposite $=$ hypotenuse $\times \sin 30^\circ$.

In (b), the triangle is half of an equilateral triangle with side 8 cm, which explains why the side opposite 30° is half the hypotenuse. The third side is $\sqrt{64 - 16} = 4\sqrt{3}$ cm.

Answer to Question 5 (4 marks)

a. Base diagonal $= \sqrt{64 + 36} = 10$ (1); space diagonal $= \sqrt{100 + 16} =$ **10.77 cm** (1) The longest rod is the space diagonal. First find the base diagonal with Pythagoras, then use Pythagoras again with the height 4 cm.

b. $\tan x = \frac{4}{10}$ (1), 21.8° (1) The height 4 cm is opposite the angle and the base diagonal 10 cm is adjacent, so we use tangent.

Check (a) in one step: $\sqrt{8^2 + 6^2 + 4^2} = \sqrt{116}$.