Worksheet - Pythagoras and trigonometry in right-angled triangles

Year: Year 11 · 20

Show your working.

Question 1

Question 1 – Finding the hypotenuse *(Section A · Skills practice)*

Find the hypotenuse of each right-angled triangle. Give non-exact answers to 2 decimal places.

a. Shorter sides 9 cm and 12 cm. [2 marks]
b. Shorter sides 7 cm and 24 cm. [2 marks]
c. Shorter sides 5 cm and 8 cm. [2 marks]
d. What do you notice about the answers to (a) and (b)? [1 mark]

Question 2

Question 2 – Finding a shorter side *(Section A · Skills practice)*

Find the missing side in each right-angled triangle.

a. Hypotenuse 17 cm, one side 8 cm. [2 marks]
b. Hypotenuse 10 cm, one side 7 cm. Give your answer to 2 decimal places. [2 marks]
c. Hypotenuse c, sides 1.5 m and 2 m. Find c. [2 marks]

Question 3

Question 3 – Is it a right-angled triangle? *(Section A · Skills practice)*

A triangle is right-angled only if $a^2 + b^2 = c^2$ for its longest side c.

a. Is the triangle with sides 20 cm, 21 cm and 29 cm right-angled? [3 marks]
b. Is the triangle with sides 8 cm, 10 cm and 13 cm right-angled? [3 marks]
c. A builder checks a corner using 3 m, 4 m and 5 m. Explain why this works. [1 mark]

Question 4

Question 4 – Finding a side using trigonometry *(Section A · Skills practice)*

Give each answer to 2 decimal places.

a. A right-angled triangle has a hypotenuse of 12 cm and an angle of 35°. Find the

side opposite the angle. [3 marks]
b. Find the side adjacent to the 35° angle. [2 marks]
c. The side adjacent to a 28° angle is 9 cm. Find the hypotenuse. [3 marks]

Question 5

Question 5 – Finding an angle *(Section A · Skills practice)*

Find the marked angle x in each right-angled triangle, to 1 decimal place.

a. Opposite 7 cm, adjacent 10 cm. [3 marks]
b. Opposite 5 cm, hypotenuse 9 cm. [3 marks]
c. Adjacent 4 cm, hypotenuse 11 cm. [3 marks]

Question 6

Question 6 – Exact trigonometric values *(Section A · Skills practice)*

Without a calculator, use the exact values you know.

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

a. Write the exact value of $\tan 60^\circ$. [2 marks]
b. A right-angled triangle has a 30° angle and hypotenuse 10 cm. Find the exact opposite and adjacent sides. [3 marks]
c. Find the exact area of the triangle. [2 marks]

Question 7

Question 7 – A ladder against a wall *(Section B · Real-life practice)*

A 5 m ladder makes an angle of 70° with the horizontal ground.

a. How high up the wall does it reach? [3 marks]
b. How far is the foot from the wall? [2 marks]
c. For safety the angle must be between 65° and 75°. Is it safe? [1 mark]
d. The ladder is moved so that its foot is 1 m from the wall. What angle does it now make with the ground? [3 marks]

Question 8

Question 8 – Height of a tree *(Section B · Real-life practice)*

Mia stands 15 m from a tree. Her eyes are 1.6 m above the ground. The angle of elevation of the top of the tree is 40°.

a. Sketch the situation and find the height of the tree above her eye level. [3 marks]
b. Find the total height of the tree. [1 mark]
c. How far is Mia's eye from the top of the tree? [2 marks]
d. Mia walks 10 m closer. What is the new angle of elevation? [3 marks]

Question 9

Question 9 – An aeroplane landing *(Section B · Real-life practice)*

A plane at a height of 3 000 m starts its descent towards the runway. The angle of depression of the runway from the plane is 6°.

a. Explain why the angle of elevation of the plane from the runway is also 6°. [1 mark]
b. Find the horizontal distance to the runway. Give your answer in km to 1 decimal place. [3 marks]
c. Find the distance flown along the descent path in km. [2 marks]
d. The plane flies at 250 km/h. How many minutes does the descent take? [2 marks]

Question 10

Question 10 – Walking north and east *(Section B · Real-life practice)*

Priya walks 8 km north and then 6 km east.

a. How far is she from the start in a straight line? [2 marks]
b. Find the bearing of her position from the start, to the nearest degree. [3 marks]
c. What is the bearing of the start from her? [1 mark]
d. She walks back directly at 4 km/h. How long does she take? [2 marks]

Question 11

Question 11 – A roof truss *(Section B · Real-life practice)*

A roof is an isosceles triangle with a horizontal span of 8 m and a height of 2.5 m at the centre.

a. Find the length of each sloping side to 2 decimal places. [3 marks]
b. Find the angle each side makes with the horizontal. [3 marks]
c. The roof is covered with tiles costing £25 per m² and the building is 12 m long. Find the cost for both sloping sides. [3 marks]

Question 12

Question 12 – A television screen *(Section B · Real-life practice)*

A television screen has a diagonal of 55 inches. The width and height are in the ratio $16 : 9$.

a. Let the width be $16k$ and the height $9k$. Show that $k^2 = \frac{55^2}{337}$. [3 marks]
b. Find the width and height to 1 decimal place. [3 marks]
c. Find the area of the screen in square inches (nearest whole number). [2 marks]

Question 13

Question 13 – A steep road *(Section B · Real-life practice)*

A road rises 1 m for every 8 m travelled horizontally.

a. Find the angle the road makes with the horizontal to 1 decimal place. [2 marks]
b. The road climbs 25 m in height. How far horizontally does it go? [1 mark]
c. How long is the road itself? [3 marks]
d. Express the gradient as a percentage. [1 mark]

Question 14

Question 14 – A ski lift *(Section B · Real-life practice)*

A ski lift cable is 600 m long and rises 180 m.

a. Find the angle of the cable to the horizontal. [2 marks]
b. Find the horizontal distance covered. [2 marks]
c. The lift travels at 2.5 m/s along the cable. How many minutes does a ride take? [2 marks]
d. A second cable is parallel and 100 m long. How high does it rise? [2 marks]

Question 15

Question 15 – Space diagonal of a cuboid (Higher) *(Section C · Problem solving)*

A cuboid measures 8 cm by 6 cm by 5 cm. $ABCD$ is the base (8 by 6) and E is above A with $AE = 5$ cm.

a. Find the diagonal AC of the base. [2 marks]
b. Find the space diagonal EC to 2 decimal places. [3 marks]
c. Find the angle EC makes with the base, to 1 decimal place. [3 marks]
d. Show that $EC^2 = 8^2 + 6^2 + 5^2$. [2 marks]

Question 16

Question 16 – A square-based pyramid (Higher) *(Section C · Problem solving)*

A pyramid has a square base of side 10 cm and a vertical height of 12 cm above the centre of the base.

a. Find the length of a half-diagonal of the base. [2 marks]

b. Find the length of a sloping edge to 2 decimal places. [3 marks]
c. Find the angle a sloping edge makes with the base. [3 marks]
d. Find the slant height of a triangular face and its area. [4 marks]

Question 17

Question 17 – A tower seen from two points (Higher) *(Section C · Problem solving)*

From a point A on level ground the angle of elevation of the top of a tower is 25°. From a point B, 40 m closer to the tower in a straight line, the angle of elevation is 40°.

a. Let h be the height of the tower. Write the distance from A and from B to the base in terms of h. [2 marks]
b. Show that $\dfrac{h}{\tan 25^\circ} - \dfrac{h}{\tan 40^\circ} = 40$. [2 marks]
c. Solve it to find h to 1 decimal place. [3 marks]

Question 18

Question 18 – Exact values and surds (Higher) *(Section C · Problem solving)*

An equilateral triangle has sides 6 cm.

a. Find the exact height of the triangle. [3 marks]
b. Find the exact area. [2 marks]
c. Show that $\sin 60^\circ = \dfrac{\sqrt{3}}{2}$ using your triangle. [2 marks]
d. A right-angled isosceles triangle has equal sides 1. Find the exact value of $\sin 45^\circ$. [3 marks]

Question 19

Question 19 – Forming and solving an equation *(Section C · Problem solving)*

A right-angled triangle has sides x cm, $(x+1)$ cm and $(x+2)$ cm, where $x+2$ is the hypotenuse.

a. Show that $x^2 - 2x - 3 = 0$. [3 marks]
b. Solve the equation. [2 marks]
c. Find the area of the triangle. [2 marks]
d. Find the smallest angle to 1 decimal place. [2 marks]

Question 20

Question 20 – Challenge: generating Pythagorean triples *(Section D · Challenge)*

For whole numbers $m > n$, a triple is $(m^2 - n^2,\ 2mn,\ m^2 + n^2)$.

a. Use $m = 5$ and $n = 2$ to find a triple and verify it. [3 marks]
b. Prove that $(m^2 - n^2)^2 + (2mn)^2 = (m^2 + n^2)^2$. [4 marks]
c. Find a triple where the hypotenuse is 25. [3 marks]