Answers - Pythagoras and trigonometry in right-angled triangles

Year: Year 11

Answer to Question 1

a. $\sqrt{81+144}=\sqrt{225}=$ **15 cm**. The hypotenuse is the longest side, so we add the squares of the two shorter sides and take the square root.
b. $\sqrt{49+576}=\sqrt{625}=$ **25 cm**. Square both shorter sides, add, then take the square root: $\sqrt{625}=25$.
c. $\sqrt{25+64}=\sqrt{89}=$ **9.43 cm**. 89 is not a square number, so the answer is not a whole number; round to 2 decimal places.
d. Both answers are whole numbers, so $(9, 12, 15)$ and $(7, 24, 25)$ are Pythagorean triples. A Pythagorean triple is a set of three whole numbers that fit $a^2+b^2=c^2$.

Answer to Question 2

a. $\sqrt{289-64}=\sqrt{225}=$ **15 cm**. To find a shorter side, subtract the square of the known side from the square of the hypotenuse.
b. $\sqrt{100-49}=\sqrt{51}=$ **7.14 cm**. Subtract the squares, then take the square root and round to 2 decimal places.
c. $\sqrt{2.25+4}=\sqrt{6.25}=$ **2.5 m**. Here both shorter sides are known, so we add their squares. The 1.5, 2, 2.5 triangle is the 3-4-5 triangle halved.

Answer to Question 3

a. $20^2+21^2=400+441=841$ and $29^2=841$. **Yes**. Square the two shorter sides and add them, then compare with the square of the longest side. They are equal, so the triangle is right-angled.
b. $8^2+10^2=164$ but $13^2=169$. **No**. The sum of the squares of the shorter sides does not equal the square of the longest side, so the triangle is not right-angled.
c. $3^2+4^2=25=5^2$, so the converse of Pythagoras' theorem shows the angle is 90°. A 3-4-5 triangle always has a right angle, so the builder knows the corner is square.

Answer to Question 4

a. $\sin 35^\circ = \dfrac{\text{opp}}{12}$, so opp $= 12 \sin 35^\circ =$ **6.88 cm**. The hypotenuse is known and we need the opposite side, so we use sine (SOH).
b. adj $= 12 \cos 35^\circ =$ **9.83 cm**. Now we need the adjacent side with the hypotenuse, so we use cosine (CAH).
c. $\cos 28^\circ = \dfrac{9}{h}$, so $h = \dfrac{9}{\cos 28^\circ} =$ **10.19 cm**. The adjacent side is known and we need the hypotenuse, so we use cosine and rearrange: divide 9 by $\cos 28^\circ$.

Answer to Question 5

a. $\tan x = \dfrac{7}{10}$, so $x = 35.0^\circ$. Opposite and adjacent are known, so we use tangent and then the inverse tan key: $\tan^{-1}(0.7)$.
b. $\sin x = \dfrac{5}{9}$, so $x = 33.7^\circ$. Opposite and hypotenuse are known, so we use sine and then $\sin^{-1}$.
c. $\cos x = \dfrac{4}{11}$, so $x = 68.7^\circ$. Adjacent and hypotenuse are known, so we use cosine and then $\cos^{-1}$.

Answer to Question 6

a. $\tan 60^\circ = \dfrac{\sin 60^\circ}{\cos 60^\circ} = \dfrac{\sqrt{3}/2}{1/2} = \sqrt{3}$. Divide the exact values from the table: dividing by $\frac{1}{2}$ is the same as multiplying by 2.
b. Opposite $= 10 \sin 30^\circ =$ **5 cm**. Adjacent $= 10 \cos 30^\circ = 5\sqrt{3}$ **cm** ≈ 8.66. Use $\sin 30^\circ = \frac{1}{2}$ for the opposite side and $\cos 30^\circ = \frac{\sqrt{3}}{2}$ for the adjacent side.
c. Area $= \frac{1}{2} \times 5 \times 5\sqrt{3} = \dfrac{25\sqrt{3}}{2}$ **cm²** ≈ 21.65 cm². The two shorter sides are perpendicular, so they are the base and the height.

Answer to Question 7

a. $5 \sin 70^\circ =$ **4.70 m**. The ladder is the hypotenuse and the height is opposite the 70° angle, so we use sine.
b. $5 \cos 70^\circ =$ **1.71 m**. The distance from the wall is adjacent to the angle, so we use cosine.
c. Yes, 70° is between 65° and 75°. So the ladder is at a safe angle.
d. $\cos x = \dfrac{1}{5}$, so $x = 78.5^\circ$ (too steep). The adjacent side is 1 m and the hypotenuse is 5 m, so we use $\cos^{-1}\left(\frac{1}{5}\right)$. This is more than 75°, so it is not safe.

Answer to Question 8

a. $15\tan 40^\circ = $ **12.59 m**. The 15 m distance is adjacent to the 40° angle and the height is opposite, so we use tangent.

b. $12.59 + 1.6 = $ **14.19 m**. Add Mia's eye height to the height above her eyes.

c. $\dfrac{15}{\cos 40^\circ} = $ **19.58 m**. The line of sight is the hypotenuse, so divide the adjacent side by $\cos 40^\circ$.

d. Now 5 m away: $\tan x = \dfrac{12.59}{5}$, so $x = 68.3^\circ$. Use tangent again with the new adjacent side $15 - 10 = 5$ m.

Answer to Question 9

a. Alternate angles between parallel lines are equal. The horizontal line at the plane and the ground are parallel, so the angle of depression and the angle of elevation are alternate angles.

b. $\dfrac{3000}{\tan 6^\circ} = 28\,543$ m $=$ **28.5 km**. The height is opposite the 6° angle and the horizontal distance is adjacent, so divide by $\tan 6^\circ$.

c. $\dfrac{3000}{\sin 6^\circ} = 28\,700$ m $=$ **28.7 km**. The descent path is the hypotenuse, so divide the height by $\sin 6^\circ$.

d. $\dfrac{28.7}{250} \times 60 = $ **6.9 minutes**. Time $=$ distance $\div$ speed, then multiply by 60 to change hours into minutes.

Answer to Question 10

a. $\sqrt{64 + 36} = $ **10 km**. North and east are at right angles, so we use Pythagoras.

b. $\tan x = \dfrac{6}{8}$, so $x = 36.9^\circ$. The bearing is 037°. The bearing is measured clockwise from north; the east distance is opposite the angle at the start and the north distance is adjacent.

c. $37 + 180 = 217^\circ$ (using 36.9°, about 217°). A back bearing differs from the bearing by 180°.

d. $10 \div 4 = $ **2.5 hours**. Time $=$ distance $\div$ speed.

Answer to Question 11

a. Half the span is 4 m: $\sqrt{4^2 + 2.5^2} = \sqrt{22.25} = $ **4.72 m**.

b. $\tan x = \frac{2.5}{4}$, so $x = 32.0°$.
c. Area of one side $= 4.72 \times 12 = 56.6$ m². Both sides $= 113.2$ m². Cost $= 113.2 \times 25 =$ **£2 830** (about).

Answer to Question 12

a. $(16k)^2 + (9k)^2 = 55^2$, so $256k^2 + 81k^2 = 337k^2 = 3025$ and $k^2 = \frac{3025}{337}$.
b. $k = 2.996$. Width $= 16k =$ **47.9 in**, height $= 9k =$ **27.0 in**.
c. $47.94 \times 26.96 =$ **1 293 square inches**. Area = width × height, using the unrounded values.

Answer to Question 13

a. $\tan x = \frac{1}{8}$, so $x = 7.1°$. The rise is opposite the angle and the horizontal distance is adjacent, so we use tangent.
b. $25 \times 8 =$ **200 m**. Every metre of height needs 8 m horizontally.
c. $\sqrt{25^2 + 200^2} =$ **201.6 m**. The road is the hypotenuse of a right-angled triangle with sides 25 m and 200 m.
d. $\frac{1}{8} =$ **12.5%**. The gradient is the rise divided by the horizontal distance, then multiplied by 100.

Answer to Question 14

a. $\sin x = \frac{180}{600} = 0.3$, so $x = 17.5°$. The rise is opposite the angle and the cable is the hypotenuse, so we use sine.
b. $\sqrt{600^2 - 180^2} =$ **572.4 m**. Use Pythagoras to find the horizontal side.
c. $600 \div 2.5 = 240$ s $=$ **4 minutes**. Time = distance ÷ speed, then divide by 60 to change seconds into minutes.
d. Similar triangles: $100 \times 0.3 =$ **30 m**. The cables are at the same angle, so the rise is always 0.3 of the length.

Answer to Question 15

a. $AC = \sqrt{64 + 36} =$ **10 cm**. The base is a rectangle, so its diagonal is the hypotenuse of a right-angled triangle with sides 8 cm and 6 cm.

b. $EC = \sqrt{10^2 + 5^2} = \sqrt{125} =$ **11.18 cm**. Triangle EAC has a right angle at A, with sides $AC = 10$ and $AE = 5$.
c. $\tan x = \dfrac{5}{10}$, so $x = 26.6°$. The angle with the base is at C: AE is opposite and AC is adjacent.
d. $EC^2 = AC^2 + 5^2 = (8^2 + 6^2) + 5^2 = 64 + 36 + 25 = 125$. This is the 3D version of Pythagoras.

Answer to Question 16

a. Diagonal $= \sqrt{200}$, so half is $\sqrt{50} =$ **7.07 cm**.
b. $\sqrt{50 + 144} = \sqrt{194} =$ **13.93 cm**.
c. $\tan x = \dfrac{12}{7.07}$, so $x = 59.5°$.
d. The slant height uses half the side (5 cm): $\sqrt{25 + 144} =$ **13 cm**. Area $= \frac{1}{2} \times 10 \times 13 =$ **65 cm²**.

Answer to Question 17

a. From A: $\dfrac{h}{\tan 25°}$. From B: $\dfrac{h}{\tan 40°}$.
b. A is 40 m further away than B, so the difference of the two distances is 40.
c. $h(2.1445 - 1.1918) = 40$, so $0.9527h = 40$ and $h = 42.0$ **m**. Take out h as a common factor, then divide by the bracket.

Answer to Question 18

a. The height splits the base: $h = \sqrt{6^2 - 3^2} = \sqrt{27} = 3\sqrt{3}$ **cm**.
b. Area $= \frac{1}{2} \times 6 \times 3\sqrt{3} = 9\sqrt{3}$ **cm²** $= 15.59$ cm².
c. In the half triangle: $\sin 60° = \dfrac{3\sqrt{3}}{6} = \dfrac{\sqrt{3}}{2}$.
d. Hypotenuse $= \sqrt{1 + 1} = \sqrt{2}$, so $\sin 45° = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$.

Answer to Question 19

a. $x^2 + (x + 1)^2 = (x + 2)^2$, so $2x^2 + 2x + 1 = x^2 + 4x + 4$, giving $x^2 - 2x - 3 = 0$.
b. $(x - 3)(x + 1) = 0$, so $x = 3$ (a length cannot be negative).
c. Sides 3, 4, 5. Area $= \frac{1}{2} \times 3 \times 4 =$ **6 cm²**.

d. The smallest angle is opposite 3: $\sin x = \frac{3}{5}$, so $x = 36.9°$.

Answer to Question 20

a. $m^2 - n^2 = 21$, $2mn = 20$, $m^2 + n^2 = 29$. $21^2 + 20^2 = 441 + 400 = 841 = 29^2$. ✓
b. LHS $= m^4 - 2m^2n^2 + n^4 + 4m^2n^2 = m^4 + 2m^2n^2 + n^4 = (m^2 + n^2)^2$.
c. Take $m = 4$, $n = 3$: $m^2 + n^2 = 25$, and $m^2 - n^2 = 7$, $2mn = 24$. The triple is $(7, 24, 25)$.