Worksheet - Problem solving and GCSE exam practice

Year: Year 11 · 20

Show your working.

Question 1

Question 1 – Sharing in a ratio *(Section A · Skills practice)*

Three friends share £240 in the ratio $2 : 3 : 7$.

a. Work out the value of one part. [2 marks]
b. Work out how much each friend receives. [2 marks]
c. The friend with the largest share spends 35% of it. How much is that? [2 marks]

Question 2

Question 2 – Standard form *(Section A · Skills practice)*

Do not use a calculator for part (a).

a. Work out $(3 \times 10^5) \times (4 \times 10^{-2})$ and give the answer in standard form. [2 marks]
b. Write 0.00072 in standard form. [1 mark]
c. Work out $\dfrac{6 \times 10^8}{3 \times 10^3}$. [2 marks]
d. The distance to the Sun is 1.5×10^8 km. Light travels at 3×10^5 km/s. How many seconds does light take to arrive? [3 marks]

Question 3

Question 3 – Equations *(Section A · Skills practice)*

Solve each problem, showing your working.

a. $3(2x - 5) = x + 10$ [3 marks]
b. Solve the simultaneous equations $2x + y = 11$ and $x - y = 1$. [3 marks]
c. Check your answers to (b). [1 mark]

Question 4

Question 4 – A ladder against a wall *(Section A · Skills practice)*

A ladder 6.5 m long leans against a vertical wall. The foot of the ladder is 2.5 m from the wall.

a. How high up the wall does the ladder reach? [3 marks]
b. The foot is moved 1 m further from the wall. How far down does the top slide? Give your answer to 2 decimal places. [4 marks]

Question 5

Question 5 – Successive percentage changes *(Section A · Skills practice)*

A coat costs £80. In a sale the price is reduced by 15%. At the end of the sale it is reduced by a further 10% of the sale price.

a. Work out the final price. [3 marks]
b. What is the overall percentage reduction? [2 marks]
c. Tom says the reduction is 25%. Explain his mistake. [1 mark]

Question 6

Question 6 – Proof by algebra *(Section A · Skills practice)*

Prove each statement.

a. The sum of three consecutive whole numbers is always a multiple of 3. [3 marks]
b. $(n+1)^2 - (n-1)^2 = 4n$ for every number n. [3 marks]
c. Use (b) to find $51^2 - 49^2$ without a calculator. [1 mark]

Question 7

Question 7 – Choosing a phone contract *(Section B · Real-life practice)*

Contract A costs £60 up front and £15 per month. Contract B costs £22 per month with nothing up front. Both last 24 months.

a. Work out the total cost of each contract. [3 marks]
b. How much does Contract A save? [1 mark]
c. After how many months is the total cost the same? [3 marks]
d. Which contract is cheaper after 6 months? [1 mark]

Question 8

Question 8 – Scaling a recipe *(Section B · Real-life practice)*

A recipe for 4 people uses 300 g of flour and 2 eggs. Flour costs £1.20 per kilogram and eggs cost 30p each.

a. How much flour and how many eggs are needed for 10 people? [3 marks]
b. Work out the cost of the flour for 10 people. [2 marks]
c. Work out the total cost of the flour and eggs for 10 people. [2 marks]
d. Zara has 2 kg of flour. For how many people can she make the recipe? [2 marks]

Question 9

Question 9 – Lawn and pond *(Section B · Real-life practice)*

A rectangular lawn is 12 m by 8 m. A circular pond of radius 2 m is in the middle. Grass seed costs £6.50 per bag and one bag covers 20 m².

a. Work out the area of grass to be sown, to 2 decimal places. [3 marks]
b. How many bags are needed? [2 marks]
c. Work out the total cost. [1 mark]
d. The pond edging costs £4.20 per metre. Work out the cost of edging the pond. [3 marks]

Question 10

Question 10 – A two-part journey *(Section B · Real-life practice)*

A van travels 90 km at 60 km/h and then 60 km at 40 km/h.

a. Work out the time for each part. [2 marks]
b. Work out the total time. [1 mark]
c. Work out the average speed for the whole journey. [2 marks]
d. Explain why the average speed is not 50 km/h simply because $(60 + 40) \div 2 = 50$. [2 marks]

Question 11

Question 11 – Counters in a bag *(Section B · Real-life practice)*

A bag contains 3 red and 7 blue counters. Two are taken at random without replacement.

a. Find the probability that both are blue. [2 marks]
b. Find the probability that they are different colours. [3 marks]
c. Find the probability that at least one is red. [2 marks]

Question 12

Question 12 – A new mean *(Section B · Real-life practice)*

The mean of five numbers is 12. A sixth number is added and the mean becomes 13.

a. Work out the total of the first five numbers. [1 mark]
b. Work out the sixth number. [2 marks]
c. A seventh number is added and the mean stays 13. Work out the seventh number. [2 marks]

Question 13

Question 13 – A wheelchair ramp *(Section B · Real-life practice)*

A ramp 5 m long makes an angle of 12° with the horizontal.

a. Work out the height of the top of the ramp. [2 marks]
b. Work out the horizontal distance covered by the ramp. [2 marks]
c. Regulations say the angle must not exceed 5°. What is the least length of ramp needed to climb 1.04 m? [3 marks]

Question 14

Question 14 – Saving for a deposit *(Section B · Real-life practice)*

Hugo invests £2 500 at 3% compound interest per year.

a. Work out the value after 4 years. [3 marks]
b. He needs £3 000. How many whole years must he wait? [3 marks]
c. Explain why simple interest at the same rate would give less after 4 years. [2 marks]

Question 15

Question 15 – An algebra problem *(Section C · Problem solving)*

A rectangle has length $(w + 5)$ cm and width w cm. Its area is 84 cm².

a. Show that $w^2 + 5w - 84 = 0$. [2 marks]
b. Solve the equation by factorising. [3 marks]
c. Work out the perimeter of the rectangle. [2 marks]
d. A square has the same perimeter. Work out its area. [3 marks]

Question 16

Question 16 – Darts in a circle *(Section C · Problem solving)*

A circle of radius 5 cm just fits inside a square. A dart is thrown at random and lands in the square.

a. Work out the area of the square. [1 mark]
b. Work out the shaded area between the circle and the square, to 2 decimal places. [3 marks]
c. Find the probability that the dart lands in the shaded area. [2 marks]
d. Estimate how many of 200 darts land in the shaded area. [2 marks]

Question 17

Question 17 – Similar containers *(Section C · Problem solving)*

Two similar cans have heights 10 cm and 15 cm. The larger can holds 675 ml.

a. Write down the length scale factor from the small can to the large can. [1 mark]
b. Work out the volume of the smaller can. [3 marks]
c. The label on the small can has area 160 cm². Work out the label area of the large can. [3 marks]

Question 18

Question 18 – Tickets and simultaneous equations *(Section C · Problem solving)*

A cinema sells adult tickets for a pounds and child tickets for c pounds. 3 adults and 2 children pay £37. 2 adults and 3 children pay £33.

a. Write two equations. [2 marks]
b. Solve them. [4 marks]
c. A group of 4 adults and 5 children goes. Work out the cost. [2 marks]

Question 19

Question 19 – Proof with an odd number *(Section C · Problem solving)*

n is a positive whole number.

a. Show that $(2n+1)^2 - 1 = 4n(n+1)$. [3 marks]
b. Explain why $n(n+1)$ is always even. [2 marks]
c. Hence prove that the square of any odd number, minus 1, is a multiple of 8. [3 marks]

Question 20

Question 20 – Challenge: melting a cylinder *(Section D · Challenge)*

A solid metal cylinder has radius 3 cm and height 10 cm. It is melted and made into identical spheres each of radius 1.5 cm. No metal is lost.

a. Work out the volume of the cylinder in terms of π. [2 marks]
b. Work out the volume of one sphere in terms of π. [2 marks]
c. How many spheres can be made? [2 marks]
d. The surface area of one sphere is $4\pi r^2$. Compare the total surface area of all the spheres with the surface area of the cylinder (including both ends). [4 marks]