Answers - Problem solving and GCSE exam practice

Year: Year 11

Answer to Question 1

a. $2 + 3 + 7 = 12$ parts, so one part is $240 \div 12 =$ **£20**. Add the numbers in the ratio to find the total number of equal parts.
b. $2 \times 20 =$ **£40**, $3 \times 20 =$ **£60**, $7 \times 20 =$ **£140**. Check: $40 + 60 + 140 = 240$. Multiply each number in the ratio by the value of one part.
c. $0.35 \times 140 =$ **£49**. 35% as a decimal is 0.35. The friend spends £49.

Answer to Question 2

a. $3 \times 4 = 12$ and $10^5 \times 10^{-2} = 10^3$, so $12 \times 10^3 = 1.2 \times 10^4$. Multiply the numbers and add the powers of 10. 12 is not between 1 and 10, so we write $12 = 1.2 \times 10^1$.
b. 7.2×10^{-4}. Move the decimal point 4 places to the right to get 7.2, so the power is -4.
c. $\frac{6}{3} = 2$ and $10^{8-3} = 10^5$: 2×10^5. Divide the numbers and subtract the powers of 10.
d. $\frac{1.5 \times 10^8}{3 \times 10^5} = 0.5 \times 10^3 =$ **500 seconds**. Time = distance ÷ speed. 0.5 is not between 1 and 10, so $0.5 \times 10^3 = 5 \times 10^2 = 500$.

Answer to Question 3

a. $6x - 15 = x + 10$, so $5x = 25$ and $x = 5$. Expand the bracket, then subtract x and add 15 to both sides. Check: $3 \times 5 = 15$ and $5 + 10 = 15$.
b. Add the equations: $3x = 12$, so $x = 4$. Then $y = 4 - 1 = 3$. $x = 4$**,** $y = 3$. The y terms have opposite signs, so adding the equations eliminates y.
c. $2 \times 4 + 3 = 11$ ✓ and $4 - 3 = 1$ ✓. Both equations are true, so the solution is correct.

Answer to Question 4

a. $h^2 = 6.5^2 - 2.5^2 = 42.25 - 6.25 = 36$, so $h =$ **6 m**. The ladder is the hypotenuse of a right-angled triangle, so we subtract the squares. The ladder reaches 6 m up

the wall.

b. New foot distance 3.5 m: $h^2 = 42.25 - 12.25 = 30$, $h = 5.48$ m. The top slides down $6 - 5.48 =$ **0.52 m**. Use Pythagoras again with the new distance 3.5 m, then subtract the new height from the old height.

Answer to Question 5

a. $80 \times 0.85 \times 0.9 =$ **£61.20**. A 15% reduction gives the multiplier 0.85 and a 10% reduction gives 0.9. Multiply by each in turn.

b. $0.85 \times 0.9 = 0.765$, so the reduction is **23.5%**. Multiply the multipliers; the part below 1 is the overall reduction.

c. He added 15% and 10%, but the second reduction is 10% of a smaller price. The overall reduction is 23.5%, not 25%.

Answer to Question 6

a. Let the numbers be n, $n+1$, $n+2$. The sum is $3n + 3 = 3(n+1)$, which is a multiple of 3. Consecutive numbers differ by 1, so we call them n, $n+1$ and $n+2$. Since $n+1$ is a whole number, the sum is 3 times a whole number.

b. $(n+1)^2 - (n-1)^2 = (n^2 + 2n + 1) - (n^2 - 2n + 1) = 4n$. Expand both squares; the n^2 terms and the 1s cancel, leaving $2n + 2n$.

c. $n = 50$: $4 \times 50 =$ **200** (check: $2601 - 2401 = 200$). $51 = 50 + 1$ and $49 = 50 - 1$, so this is the expression from part (b) with $n = 50$.

Answer to Question 7

a. A: $60 + 24 \times 15 = 60 + 360 =$ **£420**. B: $24 \times 22 =$ **£528**.

b. $528 - 420 =$ **£108**.

c. $60 + 15m = 22m$, so $60 = 7m$ and $m = 8.57$: the totals are equal after about **8.6 months**, so from month 9 A is cheaper.

d. At 6 months A costs $60 + 90 = 150$ and B costs 132, so **B** is cheaper.

Answer to Question 8

a. Multiply by $\frac{10}{4} = 2.5$: flour $300 \times 2.5 =$ **750 g**; eggs $2 \times 2.5 =$ **5 eggs**. 10 people is $\frac{10}{4} = 2.5$ times as many as 4 people, so every amount is multiplied by 2.5.

b. $0.75 \times 1.20 =$ **90p**. Change 750 g into 0.75 kg before multiplying by the price per kilogram.

c. $90 + 5 \times 30 = 90 + 150 =$ **£2.40**. Add the cost of the flour and the cost of 5 eggs.

d. $2000 \div 300 \times 4 = 26.7$, so **26 people**. Each person needs $300 \div 4 = 75$ g of flour, and $2000 \div 75 = 26.7$. We round down because the flour would run out.

Answer to Question 9

a. $12 \times 8 - \pi \times 2^2 = 96 - 12.57 =$ **83.43 m²**. Subtract the area of the circular pond from the area of the rectangle.
b. $83.43 \div 20 = 4.17$, so **5 bags**. We round up because 4 bags cover only 80 m².
c. $5 \times 6.50 =$ **£32.50**. Multiply the number of bags by the price of one bag.
d. Circumference $= 2\pi \times 2 = 12.57$ m. Cost $= 12.57 \times 4.20 =$ **£52.78**. The edging goes round the circumference of the pond.

Answer to Question 10

a. $90 \div 60 =$ **1.5 h** and $60 \div 40 =$ **1.5 h**.
b. $1.5 + 1.5 =$ **3 hours**.
c. $150 \div 3 =$ **50 km/h**.
d. The two speeds were used for different distances. Here the distances differ, but the times happen to be equal, so the answer is 50; if the times had differed, simply averaging the speeds would be wrong.

Answer to Question 11

a. $\frac{7}{10} \times \frac{6}{9} = \frac{42}{90} = \frac{7}{15}$. Multiply along the branches; the second fraction has denominator 9 because the counter is not replaced.
b. $\frac{3}{10} \times \frac{7}{9} + \frac{7}{10} \times \frac{3}{9} = \frac{21}{90} + \frac{21}{90} = \frac{42}{90} = \frac{7}{15}$. Different colours can happen in two orders, so we add the two products.
c. $1 - \frac{7}{15} = \frac{8}{15}$. 'At least one red' is the complement of 'both blue', so subtract from 1.

Answer to Question 12

a. $5 \times 12 =$ **60**. The total is the mean times the number of values.
b. New total $= 6 \times 13 = 78$, so the sixth number is $78 - 60 =$ **18**. The sixth number is the difference between the two totals.
c. The total for seven numbers is $7 \times 13 = 91$, so the seventh number is $91 - 78 =$ **13**, equal to the mean.

Answer to Question 13

a. $5 \sin 12^\circ =$ **1.04 m**. The ramp is the hypotenuse and the height is opposite the

12° angle, so we use sine.

b. $5\cos 12^\circ =$ **4.89 m**. The horizontal distance is adjacent to the angle, so we use cosine.

c. $\sin 5^\circ = \dfrac{1.04}{L}$, so $L = \dfrac{1.04}{0.0872} =$ **11.9 m**. The height is opposite the angle and the ramp length is the hypotenuse. Rearrange to make L the subject.

Answer to Question 14

a. $2500 \times 1.03^4 =$ **£2 813.77**.

b. After 6 years $2500 \times 1.03^6 = 2985.13$; after 7 years $= 3074.68$. He needs **7 years**.

c. Simple interest gives £75 a year, so £2 800 after 4 years. After 1 year both give £2 575, but from year 2 the compound interest is also paid on earlier interest, so compound gives more (£2 813.77).

Answer to Question 15

a. $w(w+5) = 84$, so $w^2 + 5w - 84 = 0$. The area of a rectangle is length times width; expand and subtract 84.

b. $(w+12)(w-7) = 0$. Width cannot be negative, so $w = 7$. We need two numbers with product -84 and sum 5: 12 and -7.

c. Length $= 12$. Perimeter $= 2 \times (7+12) =$ **38 cm**. The length is $7 + 5 = 12$ cm.

d. Square side $= 38 \div 4 = 9.5$ cm. Area $= 9.5^2 =$ **90.25 cm²**. A square has four equal sides, so each side is a quarter of the perimeter.

Answer to Question 16

a. Side $= 10$ cm, so the area is **100 cm²**. The circle just fits, so the side of the square equals the diameter, 10 cm.

b. $100 - \pi \times 5^2 = 100 - 78.54 =$ **21.46 cm²**. Subtract the area of the circle from the area of the square.

c. $\dfrac{21.46}{100} =$ **0.2146**. The probability is the shaded area divided by the area of the square.

d. $200 \times 0.2146 =$ **43**. Multiply the number of darts by the probability: $200 \times 0.2146 = 42.9$, which is about 43.

Answer to Question 17

a. $\dfrac{15}{10} =$ **1.5**. Divide the corresponding heights: large ÷ small.

b. Volume scale factor $= 1.5^3 = 3.375$. $675 \div 3.375 =$ **200 ml**. For similar shapes the

volume scale factor is the cube of the length scale factor. Divide because we are going from large to small.

c. Area scale factor $= 1.5^2 = 2.25$. $160 \times 2.25 =$ **360 cm²**. The area scale factor is the square of the length scale factor. Multiply because we are going from small to large.

Answer to Question 18

a. $3a + 2c = 37$ and $2a + 3c = 33$.

b. Multiply the first by 3 and the second by 2: $9a + 6c = 111$ and $4a + 6c = 66$. Subtract: $5a = 45$, so $a = 9$. Then $27 + 2c = 37$ so $c = 5$. **Adult £9, child £5**.

c. $4 \times 9 + 5 \times 5 = 36 + 25 =$ **£61**. Substitute $a = 9$ and $c = 5$.

Answer to Question 19

a. $(2n + 1)^2 - 1 = 4n^2 + 4n + 1 - 1 = 4n^2 + 4n = 4n(n + 1)$.

b. Of two consecutive whole numbers one is even, so the product is even.

c. Since $n(n + 1) = 2k$ for some whole number k, $4n(n + 1) = 8k$, a multiple of 8. For example, $7^2 - 1 = 48 = 8 \times 6$.

Answer to Question 20

a. $\pi \times 3^2 \times 10 = 90\pi$ **cm³**.

b. $\frac{4}{3}\pi \times 1.5^3 = \frac{4}{3}\pi \times 3.375 = 4.5\pi$ **cm³**.

c. $90\pi \div 4.5\pi =$ **20 spheres**.

d. Spheres: $20 \times 4\pi \times 2.25 = 180\pi$. Cylinder: $2\pi \times 3 \times 10 + 2\pi \times 9 = 60\pi + 18\pi = 78\pi$. The spheres have more than twice the surface area.