Answers - Probability: experiments, Venn and tree diagrams

Year: Year 11

Answer to Question 1

a. The primes are 2, 3, 5 and 7, so $\frac{4}{8} = \frac{1}{2}$.

b. The multiples of 3 are 3 and 6, so $\frac{2}{8} = \frac{1}{4}$.

c. $1 - \frac{1}{8} = \frac{7}{8}$. The spinner must either land on 5 or not land on 5, so the two probabilities add up to 1.

d. $96 \times \frac{1}{8} =$ **12 times**. Expected frequency = number of trials $\times$ probability.

Answer to Question 2

a. $\frac{14}{20} =$ **0.70**, $\frac{29}{50} =$ **0.58**, $\frac{61}{100} =$ **0.61**, $\frac{118}{200} =$ **0.59**. Relative frequency = number of successes $\div$ number of trials.

b. The result for **200 drops**, 0.59, because more trials give a more reliable estimate.

c. $500 \times 0.59 =$ **295**. Multiply the number of drops by the best estimate of the probability.

Answer to Question 3

a. The table has totals from 2 to 12; for example, the first row reads 2, 3, 4, 5, 6, 7 and the last row 7, 8, 9, 10, 11, 12.

b. Six outcomes give 7: $\frac{6}{36} = \frac{1}{6}$.

c. Totals 10, 11, 12 occur 3, 2, 1 times: $\frac{6}{36} = \frac{1}{6}$.

d. The six doubles: $\frac{6}{36} = \frac{1}{6}$.

Answer to Question 4

a. Both 4, football only $12 - 4 = 8$, hockey only $10 - 4 = 6$, neither $30 - 18 = 12$. Start with the overlap: 4 pupils play both, then subtract it from each total.

b. $P(F \cap H) = \frac{4}{30} = \frac{2}{15}$. $P(F \cup H) = \frac{8+4+6}{30} = \frac{18}{30} = \frac{3}{5}$. $F \cap H$ means both sports; $F \cup H$ means at least one sport, so we add the three regions inside the circles.
c. $\frac{12}{30} = \frac{2}{5}$. Pupils who play neither sport are outside both circles.

Answer to Question 5

a. First spin: red 0.3, not red 0.7. Second spin: red 0.3, not red 0.7 on each branch. The probabilities on each pair of branches add up to 1.
b. $0.3 \times 0.3 =$ **0.09**. Multiply along the branches red then red.
c. Red then not red: $0.3 \times 0.7 = 0.21$. Not red then red: $0.7 \times 0.3 = 0.21$. Total = **0.42**.
d. $1 - P(\text{no reds}) = 1 - 0.7 \times 0.7 = 1 - 0.49 =$ **0.51**. 'At least one red' is the complement of 'no reds'.

Answer to Question 6

a. Girls: 120, of which 90 cycle and 30 do not. Boys: 80, of which 40 cycle and 40 do not. Three quarters of 120 is 90, and half of 80 is 40.
b. $90 + 40 =$ **130 pupils**. Add the girls who cycle and the boys who cycle.
c. $\frac{90}{130} = \frac{9}{13}$. The condition 'cycles' means only the 130 cyclists count, and 90 of them are girls.

Answer to Question 7

a. $60 \div 6 =$ **10 times**.
b. $\frac{13}{60} =$ **0.217**.
c. No. A fair die would give $\frac{1}{6} = 0.167$ for a 6. The result 0.217 is higher, but the frequencies (8 to 13) are all quite close to 10, and with only 60 rolls this kind of variation is normal.
d. Roll the die many more times (hundreds) and compare the relative frequencies with $\frac{1}{6}$.

Answer to Question 8

a. $0.4 + 0.35 + 0.25 = 1$. ✓ The three options cover every pupil, so the probabilities

must add up to 1.

b. $240 \times 0.4 =$ **96 pupils**. Multiply the number of pupils by the probability of pizza.

c. $0.35 \times 0.35 =$ **0.1225**. The choices are independent, so we multiply the probabilities.

d. Not salad is 0.75, so $0.75 \times 0.75 =$ **0.5625**. The probability of not salad is $1 - 0.25 = 0.75$ for each pupil.

Answer to Question 9

a. Both stages: red $\frac{3}{8}$, blue $\frac{5}{8}$. The counter is replaced, so the probabilities are the same at both stages: 3 red out of 8 and 5 blue out of 8.

b. $P(RR) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$. $P(BB) = \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$. Multiply along the branches.

c. $P(RB) + P(BR) = 2 \times \frac{3}{8} \times \frac{5}{8} = \frac{30}{64} = \frac{15}{32}$. (Check: $\frac{9}{64} + \frac{25}{64} + \frac{30}{64} = 1$.) Different colours can happen in two orders, red then blue or blue then red, so we add the two products.

Answer to Question 10

a. First: red $\frac{4}{10}$, blue $\frac{6}{10}$. After red: red $\frac{3}{9}$, blue $\frac{6}{9}$. After blue: red $\frac{4}{9}$, blue $\frac{5}{9}$. The counter is not replaced, so the second-stage fractions have denominator 9.

b. $\frac{4}{10} \times \frac{6}{9} + \frac{6}{10} \times \frac{4}{9} = \frac{24}{90} + \frac{24}{90} = \frac{48}{90} = \frac{8}{15}$. Different colours can happen in two orders, so we add the two products.

c. $1 - \frac{8}{15} = \frac{7}{15}$. (Check: $\frac{12}{90} + \frac{30}{90} = \frac{42}{90} = \frac{7}{15}$.) Same colour is the complement of different colours.

Answer to Question 11

a. French only $45 - 12 = 33$, both 12, Spanish only $35 - 12 = 23$, neither $80 - 68 = 12$. Start with the overlap of 12, then subtract it from each total.

b. $\frac{33 + 12 + 23}{80} = \frac{68}{80} = \frac{17}{20}$. $F \cup S$ means at least one language, so add the three regions inside the circles.

c. $\frac{33 + 23}{80} = \frac{56}{80} = \frac{7}{10}$. Exactly one language means French only or Spanish only, not both.

d. $\frac{12}{45} = \frac{4}{15}$. Only the 45 French students count, and 12 of them also study Spanish.

Answer to Question 12

a. Rows: A=1: 1, 2, 3; A=2: 2, 4, 6; A=3: 3, 6, 9; A=4: 4, 8, 12. There are 12 outcomes.
b. The product is odd only when both numbers are odd: A is 1 or 3 and B is 1 or 3, which is 4 outcomes. So even $= \frac{8}{12} = \frac{2}{3}$.
c. The products equal to 6 are $(2, 3)$ and $(3, 2)$: $\frac{2}{12} = \frac{1}{6}$.
d. Products above 5: 6, 6, 9, 8, 12, which is 5 outcomes: $\frac{5}{12}$.

Answer to Question 13

a. Saturday: rain 0.4, dry 0.6. Sunday: rain 0.3, dry 0.7 (on each branch). The days are independent, so the Sunday probabilities are the same on both branches.
b. $0.4 \times 0.3 =$ **0.12**. Multiply along the branches rain then rain.
c. $0.6 \times 0.7 =$ **0.42**. Dry has probability $1 - 0.4 = 0.6$ on Saturday and $1 - 0.3 = 0.7$ on Sunday.
d. $1 - 0.12 - 0.42 =$ **0.46**. (Or $0.4 \times 0.7 + 0.6 \times 0.3 = 0.28 + 0.18 = 0.46$.)

Answer to Question 14

a. $0.8^3 =$ **0.512**. The penalties are independent, so multiply 0.8 three times.
b. $1 - 0.512 =$ **0.488**. 'Misses at least one' is the complement of 'scores all three'.
c. There are three ways to score two and miss one: each has probability $0.8 \times 0.8 \times 0.2 = 0.128$. So $3 \times 0.128 =$ **0.384**.
d. $0.2^3 =$ **0.008**. Missing has probability $1 - 0.8 = 0.2$ each time.

Answer to Question 15

a. Music only $28 - x$, both x, drama only $22 - x$, neither 8. Total $= 58 - x = 50$, so $x = 8$. Each pupil is counted once in the four regions, so the regions add up to 50.
b. Music only 20, both 8, drama only 14, neither 8.
c. $\frac{20 + 14}{50} = \frac{34}{50} = \frac{17}{25}$. Exactly one activity means music only or drama only.
d. $\frac{8}{22} = \frac{4}{11}$. Only the 22 drama pupils count, and 8 of them also do music.

Answer to Question 16

a. Petrol 150 (old 45, new 105). Other 100 (old 20, new 80). 60% of 250 is 150 petrol cars, so 100 cars are diesel or electric. Then 30% of 150 is 45 and 20% of 100 is 20.
b. $45 + 20 =$ **65 cars**.
c. $\frac{45}{65} = \frac{9}{13}$. Only the 65 old cars count, and 45 of them are petrol.
d. Old petrol: 45. New non-petrol: 80. A **new non-petrol** car is more likely.

Answer to Question 17

a. $\frac{n}{5} \times \frac{n}{5} = 0.16$. The counter is replaced, so the probability of red is $\frac{n}{5}$ both times.
b. $\frac{n^2}{25} = 0.16$, so $n^2 = 4$ and $n = 2$. Multiply both sides by 25 and take the positive square root, because n is a number of counters.
c. $P(R) = \frac{2}{5}$ and $P(\text{not } R) = \frac{3}{5}$. Different: $2 \times \frac{2}{5} \times \frac{3}{5} = \frac{12}{25} = 0.48$. Different colours can happen in two orders, so we double one product.

Answer to Question 18

a. $\frac{232}{400} =$ **0.58**.
b. $400 \times 0.5 =$ **200**.
c. The coin gave 32 more heads than expected and the relative frequency 0.58 is quite far from 0.5, so the coin **might be biased**. A big difference after 400 tosses is unlikely if the coin is fair.
d. $1000 \times 0.58 =$ **580**.
e. Toss it many more times (thousands) and see if the relative frequency stays near 0.58.

Answer to Question 19

a. Mint-mint: $\frac{5}{12} \times \frac{4}{11} = \frac{20}{132}$. Toffee-toffee: $\frac{4}{12} \times \frac{3}{11} = \frac{12}{132}$. Fruit-fruit: $\frac{3}{12} \times \frac{2}{11} = \frac{6}{132}$. Total $= \frac{38}{132} = \frac{19}{66}$.
b. No fruit: $\frac{9}{12} \times \frac{8}{11} = \frac{72}{132}$. At least one fruit $= 1 - \frac{72}{132} = \frac{60}{132} = \frac{5}{11}$.
c. After a mint is taken there are 11 sweets, 3 of them fruit: $\frac{3}{11}$.

Answer to Question 20

a. HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.

b. Outcomes with at least two heads: HHH, HHT, HTH, THH. That is $\frac{4}{8} = \frac{1}{2}$.

c. At least one head removes only TTT, leaving 7 outcomes. Only HHH has three heads: $\frac{1}{7}$.

d. The extra information changes the sample space from 8 outcomes to 7, so the probability is conditional on that information.