Test answers - Iteration, growth and decay

Year: Year 11 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. 3500×1.028^3 (2) $=$ **£3 802.31** (1) 2.8% interest gives the multiplier 1.028, used once for each of the 3 years. Round to the nearest penny.
b. $3802.31 - 3500 =$ **£302.31** (1) The interest is the final amount minus the £3500 invested.

A common error is to use simple interest, $3500 \times 0.028 \times 3 = 294$, giving £3794, which earns only the method mark if the working is shown. Check: after year 1 the value is £3598, after year 2 £3698.74, and after year 3 £3802.31.

Answer to Question 2 (4 marks)

a. $24\,000 \times 0.85^4$ (2) $=$ **£12 528.15** (1) A 15% fall each year leaves 85% of the value, so the multiplier 0.85 is used 4 times.
b. $\dfrac{24000 - 12528.15}{24000} =$ **47.8%** (1) Divide the loss by the original value and multiply by 100. The van has lost 47.8% of its value.

The fall is not $4 \times 15 = 60\%$, because each year's 15% is taken from a smaller value. A quicker method for (b): $1 - 0.85^4 = 0.478$.

Answer to Question 3 (4 marks)

a. $67.20 \div 1.12 =$ **£60** (1 for the multiplier, 1 for the answer) The price after the increase is 112% of the original, so we divide by 1.12. Check: $60 \times 1.12 = 67.20$.
b. $67.20 \times 0.88 = £59.14$, which is not £60 (1); the 12% is taken from a larger amount (1) The 12% reduction is worked out on £67.20, which is bigger than £60, so it takes off more than the 12% increase added on. The final price is 64p below the original.

Answer to Question 4 (4 marks)

a. **1.651**, **1.669**, **1.671** (1 each) Start with $x_0 = 1.5$ and put each answer back into the formula: $\sqrt[3]{1.5 + 3} = \sqrt[3]{4.5}$, and so on. Use the full calculator value each time.
b. $f(x) = x^3 - x - 3$: $f(1.6) < 0$ and $f(1.7) > 0$ (1) $f(1.6) = -0.504$ and $f(1.7) = 0.213$. The sign changes, so a root lies between 1.6 and 1.7.

In (a), each step uses the previous answer, so do not round between steps. The values settle near 1.672, which is the root of $x^3 = x + 3$.

Answer to Question 5 (4 marks)

a. $800 \times 1.06^3 =$ **£952.81** (2) 6% compound interest gives the multiplier 1.06, used 3 times. Hana has £952.81.
b. Brother: $800 + 3 \times 48 =$ **£944** (1); difference **£8.81** (1) Simple interest is 6% of £800, which is £48 each year. Compound interest earns £8.81 more because interest is also paid on interest.

In (a), the interest is £48 in year 1, £50.88 in year 2 and £53.93 in year 3. Simple interest is always £48.