Worksheet - Iteration, growth and decay

Year: Year 11 · 20

Show your working.

Question 1

Question 1 – Percentage multipliers *(Section A · Skills practice)*

A percentage multiplier is the number you multiply by to change an amount by a percentage. For example, an increase of 30% has multiplier 1.30.

a. Write the multiplier for an increase of 12% and for a decrease of 8%. [2 marks]
b. Write the multiplier for an increase of 2.5% and a decrease of 15%. [2 marks]
c. Increase £240 by 35% using a single multiplier. [2 marks]
d. A price of £50 is decreased by 24%. Work out the new price. [2 marks]

Question 2

Question 2 – Compound interest *(Section A · Skills practice)*

Anna invests £2 000 in an account paying 4% compound interest each year.

a. Write the multiplier for 4% interest. [1 mark]
b. Work out the amount in the account after 3 years. [3 marks]
c. Work out the total interest earned. [1 mark]
d. Write a formula for the amount after n years. [1 mark]

Question 3

Question 3 – Depreciation *(Section A · Skills practice)*

A new car costs £15 000. Its value falls by 20% each year.

a. Write the multiplier. [1 mark]
b. Work out the value after 3 years. [3 marks]
c. By how much has the car lost value? [1 mark]
d. Work out the percentage loss over the 3 years. [2 marks]

Question 4

Question 4 – Different rates each year *(Section A · Skills practice)*

Ben invests £4 000. The interest rates are 3% in the first year, 2.5% in the second year and 2% in the third year.

a. Write the multiplier for each year. [2 marks]
b. Work out the value of the investment after 3 years. [3 marks]

c. Work out the single percentage increase that gives the same final amount. [2 marks]

Question 5

Question 5 – Reverse percentages *(Section A · Skills practice)*

Work out the original amount in each case.

a. After a 20% increase, a price is £90. [2 marks]
b. After a 15% decrease, a value is £680. [2 marks]
c. After a 4% increase, a salary is £31 200. [2 marks]

Question 6

Question 6 – Using an iterative formula (Higher) *(Section A · Skills practice)*

The equation $x^3 = x + 3$ has a solution close to 1.7. It can be solved by the iteration $x_{n+1} = \sqrt[3]{x_n + 3}$ with $x_0 = 1.5$.

a. Work out x_1, x_2 and x_3 to 3 decimal places. [3 marks]
b. Show that the equation has a solution between 1.6 and 1.7. [2 marks]
c. Write the solution to 2 decimal places. [1 mark]

Question 7

Question 7 – Saving for a holiday *(Section B · Real-life practice)*

Maya puts £1 500 into a savings account that pays 4.5% compound interest per year.

a. How much is in the account after 5 years? [3 marks]
b. How much interest has she earned? [1 mark]
c. She wants at least £2 000. Does she have enough after 7 years? [3 marks]
d. Explain why compound interest gives more than simple interest. [1 mark]

Question 8

Question 8 – Rising prices *(Section B · Real-life practice)*

A loaf of bread costs £3.20. Because of inflation, prices rise by 3% each year.

a. Work out the price after 4 years, to the nearest penny. [3 marks]
b. Write a formula for the price after n years. [1 mark]
c. What is the percentage increase over the 4 years, to 1 decimal place? [2 marks]
d. A newspaper costs £1.40 today. Which will cost more after 4 years, the loaf or the newspaper? Show how you know. [2 marks]

Question 9

Question 9 – A shrinking town *(Section B · Real-life practice)*

The population of a town is 80 000. It falls by 2% each year.

a. Work out the population after 10 years to the nearest whole number. [3 marks]
b. Write the percentage fall over 10 years to 1 decimal place. [2 marks]
c. Use trial and improvement to find the first year in which the population is below 70 000. [3 marks]

Question 10

Question 10 – Bacteria in a dish *(Section B · Real-life practice)*

A scientist grows bacteria. There are 200 bacteria at the start. The number increases by 50% each hour.

a. Work out the number after 4 hours. [3 marks]
b. Write a formula for the number after t hours. [1 mark]
c. After how many whole hours is the number first more than 2 000? [3 marks]

Question 11

Question 11 – A phone loses value *(Section B · Real-life practice)*

A new phone costs £600. It loses 30% of its value in the first year and then 20% of its value each following year.

a. Work out the value after 3 years. [3 marks]
b. What is the percentage lost over 3 years? [2 marks]
c. The seller says "It has lost less than half its value." Is the seller correct? [2 marks]

Question 12

Question 12 – Is she correct? *(Section B · Real-life practice)*

A shop raises all its prices by 10%. A month later it reduces them by 10%. Maria says "The prices are back to where they started."

a. A jacket costs £100 at the start. Work out its price after the rise and after the reduction. [3 marks]
b. Is Maria correct? Explain. [2 marks]
c. Write the single multiplier for the two changes and the overall percentage change. [2 marks]

Question 13

Question 13 – Simple and compound interest *(Section B · Real-life practice)*

Two banks offer 5% interest per year on £1 000 for 10 years. Bank S pays simple interest and bank C pays compound interest.

a. Work out the amount in Bank S after 10 years. [2 marks]
b. Work out the amount in Bank C after 10 years. [2 marks]
c. How much more is earned in Bank C? [1 mark]
d. Explain why the difference grows each year. [2 marks]

Question 14

Question 14 – Doubling your money *(Section B · Real-life practice)*

Isaac invests £5 000 at 7% compound interest per year.

a. Work out the value after 10 years. [2 marks]
b. After how many whole years has the money first more than doubled? [3 marks]
c. Is it true that money doubles in 14 years at 7% simple interest? Explain. [3 marks]

Question 15

Question 15 – Working backwards from compound interest *(Section C · Problem solving)*

A sum of money has grown to £2 315.25 after 3 years at 5% compound interest per year.

a. Write the overall multiplier for the 3 years. [2 marks]
b. Work out the original sum. [3 marks]
c. Work out the amount after the first year. [2 marks]
d. Check that your answer works by growing it forward. [2 marks]

Question 16

Question 16 – Finding the rate *(Section C · Problem solving)*

£4 000 is invested at a fixed compound interest rate and grows to £4 494.40 in 2 years.

a. Show that the overall multiplier is 1.1236. [2 marks]
b. Work out the yearly percentage rate. [3 marks]
c. Check your answer. [2 marks]
d. How much would it be after 5 years at this rate? [2 marks]

Question 17

Question 17 – Radioactive decay *(Section C · Problem solving)*

A sample of 80 g of a radioactive substance loses 25% of its mass each day.

a. Work out the mass after 3 days. [2 marks]
b. Write a formula for the mass after n days. [1 mark]
c. After how many whole days is the mass first below 10 g? Show your working. [4 marks]
d. Roughly how many days does it take for the mass to halve? [2 marks]

Question 18

Question 18 – Comparing two accounts *(Section C · Problem solving)*

Bank A pays 3.2% compound interest per year. Bank B pays 2.5% in the first year and 3.5% in each of the next three years. Zara invests £1 000 for 4 years.

a. Work out the amount in Bank A. [2 marks]
b. Work out the amount in Bank B. [3 marks]
c. Which bank is better and by how much? [2 marks]
d. Work out the average yearly percentage for each bank (the single yearly rate that gives the same final amount) to 2 decimal places. [4 marks]

Question 19

Question 19 – An iteration with a golden ratio (Higher) *(Section C · Problem solving)*

The equation $x^2 - x - 1 = 0$ can be rearranged as $x = 1 + \dfrac{1}{x}$. Use the iteration $x_{n+1} = 1 + \dfrac{1}{x_n}$ with $x_0 = 1$.

a. Show that the rearrangement is correct. [2 marks]
b. Work out x_1, x_2, x_3, x_4. [3 marks]
c. Show that a root lies between 1 and 2. [2 marks]
d. Write the root to 2 decimal places and check it in the original equation. [3 marks]

Question 20

Question 20 – Challenge: the rule of 72 *(Section D · Challenge)*

A sum of money doubles in exactly 12 years with compound interest at a fixed yearly rate.

a. Write an equation for the yearly multiplier m and solve it. [3 marks]
b. Write the rate as a percentage to 1 decimal place. [1 mark]
c. A rule of thumb says that the doubling time is about $72 \div r$ years for a rate of $r\%$. Use $r = 6$ to test it. [3 marks]
d. Explain why this rule works less well at a rate of 30%. [3 marks]