Answers - Iteration, growth and decay

Year: Year 11

Answer to Question 1

a. Increase: $1 + 0.12 =$ **1.12**. Decrease: $1 - 0.08 =$ **0.92**. For an increase add the percentage (as a decimal) to 1; for a decrease subtract it from 1.
b. $1 + 0.025 =$ **1.025**; $1 - 0.15 =$ **0.85**. 2.5% as a decimal is 0.025 and 15% is 0.15.
c. $240 \times 1.35 =$ **£324**. A 35% increase gives the multiplier 1.35, so one multiplication does the whole job.
d. $50 \times 0.76 =$ **£38**. A 24% decrease leaves 76%, so the multiplier is 0.76.

Answer to Question 2

a. **1.04**. 4% as a decimal is 0.04, and we keep the original 100%, so the multiplier is $1 + 0.04 = 1.04$.
b. $2000 \times 1.04^3 = 2000 \times 1.124864 =$ **£2 249.73**. Compound interest multiplies by 1.04 once for each year, so we use the power 3. Round to the nearest penny.
c. $2249.73 - 2000 =$ **£249.73**. The interest is the final amount minus the amount invested.
d. Amount $= 2000 \times 1.04^n$. After n years the multiplier has been applied n times.

Answer to Question 3

a. $1 - 0.20 =$ **0.8**. A 20% fall leaves 80% of the value each year.
b. $15\,000 \times 0.8^3 = 15\,000 \times 0.512 =$ **£7 680**. The multiplier is applied once for each of the 3 years.
c. $15\,000 - 7\,680 =$ **£7 320**. Subtract the value after 3 years from the original price.
d. $\frac{7320}{15000} \times 100 =$ **48.8%** (not $3 \times 20 = 60\%$). Divide the loss by the original value.
The loss is not 60%, because each fall is worked out on a smaller value.

Answer to Question 4

a. Year 1: **1.03**, year 2: **1.025**, year 3: **1.02**. Add each rate (as a decimal) to 1.
b. $4000 \times 1.03 \times 1.025 \times 1.02 =$ **£4 307.46**. Multiply by each year's multiplier in turn, and round to the nearest penny.
c. Overall multiplier $= 1.03 \times 1.025 \times 1.02 = 1.076865$, so the increase is about **7.7%**. Multiply the three multipliers together; the part above 1 is the overall increase.

Answer to Question 5

a. Multiplier 1.2, so original $= 90 \div 1.2 =$ **£75**. The price after the increase is 120% of the original, so we divide by 1.2. Check: $75 \times 1.2 = 90$.
b. Multiplier 0.85, so original $= 680 \div 0.85 =$ **£800**. The value after the decrease is 85% of the original, so we divide by 0.85. Check: $800 \times 0.85 = 680$.
c. $31\,200 \div 1.04 =$ **£30 000**. The new salary is 104% of the old one, so we divide by 1.04. Check: $30\,000 \times 1.04 = 31\,200$.

Answer to Question 6

a. $x_1 = \sqrt[3]{4.5} =$ **1.651**; $x_2 = \sqrt[3]{4.651} =$ **1.669**; $x_3 = \sqrt[3]{4.669} =$ **1.671**.
b. Let $f(x) = x^3 - x - 3$. $f(1.6) = 4.096 - 1.6 - 3 = -0.504 < 0$ and $f(1.7) = 4.913 - 1.7 - 3 = 0.213 > 0$. The sign changes, so there is a solution between 1.6 and 1.7.
c. The values settle at about 1.672, so $x = 1.67$ to 2 d.p.

Answer to Question 7

a. $1500 \times 1.045^5 =$ **£1 869.27**. 4.5% interest gives the multiplier 1.045, used 5 times.
b. $1869.27 - 1500 =$ **£369.27**. Subtract the amount invested.
c. $1500 \times 1.045^7 = 1500 \times 1.36086 = 2041.29$, which is more than £2 000, so **yes**.
d. With compound interest, interest is paid on the interest already earned as well as on the original amount.

Answer to Question 8

a. $3.20 \times 1.03^4 =$ **£3.60**. 3% inflation gives the multiplier 1.03, used 4 times.
b. Price $= 3.20 \times 1.03^n$.
c. $1.03^4 = 1.1255$, so the increase is **12.6%**. Subtract 1 from the overall multiplier and change it into a percentage.
d. Both rise by the same percentage, but the loaf costs more in pounds: $1.40 \times 1.1255 = £1.58$, which is less than £3.60.

Answer to Question 9

a. $80\,000 \times 0.98^{10} =$ **65 366**. A 2% fall gives the multiplier 0.98, used 10 times.
b. $65\,366 \div 80\,000 = 0.817$, so it has fallen by about **18.3%** (not 20%).
c. After 6 years: $80\,000 \times 0.98^6 = 70\,867$. After 7 years: $80\,000 \times 0.98^7 = 69\,450$. So

the population first falls below 70 000 in **year 7**.

Answer to Question 10

a. $200 \times 1.5^4 = 200 \times 5.0625 =$ **1 012.5**, about **1 013 bacteria**. A 50% increase gives the multiplier 1.5, used 4 times. We round because we cannot have half a bacterium.
b. $N = 200 \times 1.5^t$. The number starts at 200 and is multiplied by 1.5 each hour.
c. $t = 5$: 1519. $t = 6$: 2278. So after **6 hours** the number is first more than 2 000. Try whole numbers of hours until the number first passes 2000.

Answer to Question 11

a. $600 \times 0.7 \times 0.8 \times 0.8 =$ **£268.80**. A 30% loss gives the multiplier 0.7 and each 20% loss gives 0.8. Multiply by each one in turn.
b. $\dfrac{600 - 268.80}{600} =$ **55.2%**. Divide the total loss, £331.20, by the original £600 and multiply by 100.
c. No: it has lost 55.2%, which is more than half. Half of £600 is £300, and the phone is now worth only £268.80, which is less than half.

Answer to Question 12

a. After the rise: $100 \times 1.1 =$ **£110**. After the reduction: $110 \times 0.9 =$ **£99**.
b. **No**: the jacket costs £99, which is £1 less than the start, because the 10% reduction applies to the higher price.
c. $1.1 \times 0.9 = 0.99$, so the overall change is a **1% decrease**.

Answer to Question 13

a. Simple: 5% of £1 000 is £50 a year, so $1000 + 10 \times 50 =$ **£1 500**.
b. $1000 \times 1.05^{10} =$ **£1 628.89**.
c. $1628.89 - 1500 =$ **£128.89**.
d. In compound interest each year's interest is also earned on the previous interest, so the yearly interest keeps growing, while the simple interest stays at £50 a year.

Answer to Question 14

a. $5000 \times 1.07^{10} = 5000 \times 1.9672 =$ **£9 835.76**.

b. After 10 years it is $1.967\times$ the start (not quite double). After 11 years it is $1.07^{11} = 2.105\times$, which is more than double. So **11 years**.
c. Simple interest at 7% gives £350 a year, so doubling needs £5 000 of interest: $5000 \div 350 = 14.3$ years. So in 14 years it is **not quite** doubled; it takes about 14.3 years.

Answer to Question 15

a. $1.05^3 =$ **1.157625**. 5% interest gives the multiplier 1.05, so over 3 years the multiplier is 1.05^3.
b. $2315.25 \div 1.157625 =$ **£2 000**. To reverse the growth we divide by the overall multiplier. The original sum was £2000.
c. $2000 \times 1.05 =$ **£2 100**. Multiply the original sum by 1.05 once.
d. $2100 \times 1.05 = 2205$ and $2205 \times 1.05 = 2315.25$. ✓ Growing the answer forward gives back £2315.25, so it is correct.

Answer to Question 16

a. $4494.40 \div 4000 = 1.1236$. Divide the final amount by the amount invested to get the overall multiplier.
b. The yearly multiplier is the square root: $\sqrt{1.1236} = 1.06$, so the rate is **6%**. The same multiplier is used twice, so the yearly multiplier is the square root of the overall one.
c. $4000 \times 1.06^2 = 4000 \times 1.1236 = 4494.40$. ✓ Growing £4000 by 6% for 2 years gives back the final amount, so the rate is correct.
d. $4000 \times 1.06^5 =$ **£5 352.90**. Use the multiplier 1.06 five times: $1.06^5 \approx 1.33823$.

Answer to Question 17

a. $80 \times 0.75^3 =$ **33.75 g**. A 25% loss gives the multiplier 0.75, used 3 times.
b. $M = 80 \times 0.75^n$.
c. After 7 days: $80 \times 0.75^7 = 10.68$ g (still above 10). After 8 days: $80 \times 0.75^8 = 8.01$ g. So after **8 days**.
d. $0.75^2 = 0.5625$ and $0.75^3 = 0.42$: the mass halves between 2 and 3 days, in about **2.4 days**.

Answer to Question 18

a. $1000 \times 1.032^4 =$ **£1 134.28**. 3.2% interest gives the multiplier 1.032, used 4 times.

b. $1000 \times 1.025 \times 1.035^3 =$ **£1 136.44**. Use 1.025 for the first year and 1.035 for each of the next three years.
c. **Bank B** is better by $1136.44 - 1134.28 =$ **£2.16**.
d. Bank A: **3.2%**. Bank B: $\sqrt[4]{1.13644} = 1.0325$, so about **3.25%**.

Answer to Question 19

a. $x^2 - x - 1 = 0$ gives $x^2 = x + 1$. Divide by x: $x = 1 + \frac{1}{x}$.
b. $x_1 = 1 + \frac{1}{1} =$ **2**; $x_2 = 1 + \frac{1}{2} =$ **1.5**; $x_3 = 1 + \frac{1}{1.5} =$ **1.667**; $x_4 = 1 + \frac{1}{1.667} =$ **1.6**.
c. $f(x) = x^2 - x - 1$: $f(1) = -1 < 0$ and $f(2) = 1 > 0$, so there is a sign change and a root between 1 and 2.
d. The values settle near 1.618, so $x = 1.62$. Check: $1.618^2 - 1.618 - 1 = 0.000$. ✓

Answer to Question 20

a. $m^{12} = 2$, so $m = 2^{1/12} =$ **1.0595**.
b. The rate is **5.9%** (to 1 decimal place).
c. At 6%: $72 \div 6 = 12$ years. $1.06^{12} = 2.012$, so the money has just doubled. The rule is **accurate**.
d. At 30%: the rule gives $72 \div 30 = 2.4$ years, but $1.3^3 = 2.2$ and $1.3^2 = 1.69$, so the real doubling time is about 2.6 years. The rule becomes less accurate for large rates because it is based on small percentages.