Test answers - Functions and graph transformations

Year: Year 11 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. $2 \times 16 - 3 = \mathbf{29}$ (1) Square 4 first, then multiply by 2 and subtract 3. So $f(4) = 29$.

b. $2x^2 = 18$ (1), $x^2 = 9$ (1), $x = 3$ **or** $x = -3$ (1) Add 3 to both sides, divide by 2, then take the square root, remembering both signs. Check: $2 \times 9 - 3 = 15$.

In (a), a common error is to work out $(2 \times 4)^2 - 3 = 61$: the index applies only to x, not to $2x$. In (b), giving only $x = 3$ loses the final mark, since $(-3)^2 = 9$ as well.

Answer to Question 2 (4 marks)

a. $fg(x) = 3x + 4$ and $gf(x) = 3(x + 4) = 3x + 12$ (1 each) For fg, apply g first then f. For gf, apply f first then g, and expand the bracket.

b. $3x + 4 = 3x + 12$ gives $4 = 12$, which is never true, so there is **no solution** (1); the order of the functions always matters here (1) Subtracting $3x$ from both sides leaves $4 = 12$, which is false for every x.

For example, with $x = 1$, $fg(1) = 7$ but $gf(1) = 15$.

Answer to Question 3 (4 marks)

a. $y = \dfrac{x - 5}{2}$ so $x = 2y + 5$ (1): $f^{-1}(x) = 2x + 5$ (1) Multiply both sides by 2, then add 5. Write the inverse using x.

b. $f^{-1}(7) = 19$ (1); $f(19) = \dfrac{14}{2} = 7$ ✓ (1) Substitute 7 into the inverse: $2 \times 7 + 5 = 19$. Applying f to 19 gives back 7.

In (a), another method is to reverse the steps of f: f subtracts 5 then halves, so f^{-1} doubles then adds 5. A common error is to write $f^{-1}(x) = \dfrac{2}{x - 5}$, confusing the inverse with the reciprocal.

Answer to Question 4 (4 marks)

a. $f(x - 2)$: $(5, 6)$ (1); $f(x) + 4$: $(3, 10)$ (1) $f(x - 2)$ moves the graph 2 units right, and $+4$ moves it 4 units up.

b. $(3, -6)$ (1), a **minimum** (1) $-f(x)$ reflects the graph in the x-axis, so the maximum becomes a minimum.

In (a), a common error is to move $f(x-2)$ 2 units to the left; the minus sign inside the bracket moves the graph to the right. Only the x-coordinate changes for $f(x-2)$, and only the y-coordinate changes for $f(x)+4$.

Answer to Question 5 (4 marks)

a. $(x-3)^2 - 9 + 5 = (x-3)^2 - 4$ (1 for a, 1 for b) Halve -6 to get -3, then subtract $(-3)^2 = 9$ to keep the expression the same.

b. Translation $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ (1); minimum value -4 (1) -3 inside the bracket moves $y = x^2$ 3 units right and -4 moves it 4 units down. The minimum point is $(3, -4)$.

Check (a) by expanding: $(x-3)^2 - 4 = x^2 - 6x + 9 - 4 = x^2 - 6x + 5$. The minimum occurs when $x = 3$.