Worksheet - Functions and graph transformations

Year: Year 11 · 20

Show your working.

Question 1

Question 1 – Using function notation (Higher) *(Section A · Skills practice)*

The function f is defined by $f(x) = 3x - 2$ and the function g by $g(x) = x^2 + 1$.

a. Work out $f(4)$, $f(-1)$ and $f(0.5)$. [3 marks]
b. Solve $f(x) = 19$. [2 marks]
c. Work out $g(3)$ and $g(-2)$. [2 marks]
d. Solve $g(x) = 26$. [2 marks]

Question 2

Question 2 – Composite functions (Higher) *(Section A · Skills practice)*

$f(x) = 2x + 3$ and $g(x) = x^2$.

a. Find $fg(x)$ and $gf(x)$, and simplify each. [4 marks]
b. Work out $fg(2)$ and $gf(2)$. [2 marks]
c. Solve $fg(x) = 35$. [3 marks]
d. Explain why $fg(x) \neq gf(x)$. [1 mark]

Question 3

Question 3 – Inverse functions (Higher) *(Section A · Skills practice)*

Find the inverse of each function.

a. $f(x) = 4x - 7$ [2 marks]
b. $g(x) = \dfrac{x + 2}{5}$ [2 marks]
c. $h(x) = \dfrac{x}{3} + 1$ [2 marks]
d. Use your answer to (a) to find $f^{-1}(9)$ and check that $f(f^{-1}(9)) = 9$. [2 marks]

Question 4

Question 4 – Domain and range (Higher) *(Section A · Skills practice)*

The domain of a function is the set of allowed inputs and the range is the set of outputs.

a. $f(x) = x^2$ with domain $-3 \leq x \leq 2$. Find the range. [3 marks]
b. $g(x) = \dfrac{1}{x}$. Which value of x must be excluded from the domain? [1 mark]
c. $h(x) = \sqrt{x - 1}$. Write the domain and the range. [3 marks]
d. $k(x) = 2x + 1$ with domain $x \geq 0$. What is the range? [2 marks]

Question 5

Question 5 – Transforming a point on a graph (Higher) *(Section A · Skills practice)*

The point $(3, 4)$ is on the graph of $y = f(x)$. Write down the coordinates of the image of this point on each graph.

a. $y = f(x) + 2$ [1 mark]
b. $y = f(x + 1)$ [2 marks]
c. $y = 2f(x)$ [1 mark]
d. $y = f(2x)$ [2 marks]
e. $y = -f(x)$ [1 mark]
f. $y = f(-x)$ [1 mark]

Question 6

Question 6 – Sketching standard graphs (Higher) *(Section A · Skills practice)*

Sketch each graph, marking where it crosses the axes and any turning point.

a. $y = x^2 - 4x + 3$ [4 marks]
b. $y = 2^x$ [3 marks]
c. $y = \dfrac{1}{x}$ [2 marks]

Question 7

Question 7 – Taxi fares (Higher) *(Section B · Real-life practice)*

A taxi company charges £2.50 plus £1.60 per mile. The fare for d miles is $f(d) = 2.5 + 1.6d$.

a. Work out $f(5)$. [1 mark]
b. Find the inverse function f^{-1} and explain what it tells you. [3 marks]
c. Use it to find how far you can travel for £15.70. [2 marks]
d. The company adds a 10% tip. Write the total as a composite function $g(f(d))$ where $g(x) = 1.1x$, and find the total for 5 miles. [3 marks]

Question 8

Question 8 – Changing money (Higher) *(Section B · Real-life practice)*

A bank charges a fixed fee of £3 and then converts pounds to euros at a rate of 1.15 euros per pound. Let $h(x) = x - 3$ be the amount after the fee and $g(x) = 1.15x$ the conversion.

a. Write $gh(x)$ and explain what it represents. [2 marks]
b. How many euros do you get for £200? [2 marks]
c. Find $(gh)^{-1}(x)$. [3 marks]
d. Use the inverse to find how many pounds you need to get 345 euros. [2 marks]

Question 9

Question 9 – Temperature conversion (Higher) *(Section B · Real-life practice)*

The temperature in degrees Celsius is $C(F) = \frac{5}{9}(F - 32)$ where F is the temperature in degrees Fahrenheit.

a. Work out $C(77)$. [2 marks]
b. Find the inverse function C^{-1}. [3 marks]
c. Use it to find the Fahrenheit temperature that equals 37°C. [2 marks]
d. Show that the two scales agree at -40. [2 marks]

Question 10

Question 10 – Discount and VAT (Higher) *(Section B · Real-life practice)*

A shop gives a 10% discount, $f(x) = 0.9x$, and then adds 20% VAT, $g(x) = 1.2x$. x is the price in pounds.

a. Write $gf(x)$ and simplify. [2 marks]
b. Work out the final price of a laptop with starting price £250. [1 mark]
c. Does it matter whether the VAT is added before or after the discount? Show that $fg(x) = gf(x)$. [3 marks]
d. Find $(gf)^{-1}(x)$ and use it to find the starting price when the final price is £162. [4 marks]

Question 11

Question 11 – A falling ball (Higher) *(Section B · Real-life practice)*

A ball is dropped from a height of 20 m. Its height after t seconds is $h(t) = 20 - 5t^2$ for $0 \le t \le 2$.

a. Work out $h(0)$, $h(1)$ and $h(2)$. [3 marks]
b. Write the range of h. [2 marks]
c. Solve $h(t) = 10$ to 2 decimal places. [3 marks]
d. Explain why the domain stops at $t = 2$. [1 mark]

Question 12

Question 12 – Moving a graph (Higher) *(Section B · Real-life practice)*

The graph of $y = f(x)$ has its minimum point at $(2, -3)$.

a. Write the minimum point of $y = f(x + 4)$. [2 marks]
b. Write the minimum point of $y = f(x) - 5$. [2 marks]
c. Write the minimum point of $y = 2f(x)$. [2 marks]
d. Write the minimum point of $y = f(2x)$. [2 marks]
e. The graph of $y = -f(x)$ has a turning point. Is it a maximum or a minimum? Give its coordinates. [2 marks]

Question 13

Question 13 – Exponential graphs (Higher) *(Section B · Real-life practice)*

The graph of $y = 2^x$ is drawn for $-1 \leq x \leq 3$.

a. Complete a table of values for $x = -1, 0, 1, 2, 3$. [2 marks]
b. The graph of $y = 2^x + 3$ is drawn. Write the equation of the horizontal asymptote and the y-intercept. [3 marks]
c. Describe the transformation that maps $y = 2^x$ to $y = 2^{x+1}$. [2 marks]
d. Show that $y = 2^{x+1}$ is the same as $y = 2 \times 2^x$ and explain what this means for the graph. [3 marks]

Question 14

Question 14 – Comparing gym plans (Higher) *(Section B · Real-life practice)*

Plan A costs £15 joining fee plus £8 per month: $a(m) = 15 + 8m$. Plan B costs £30 joining fee plus £5 per month: $b(m) = 30 + 5m$.

a. Solve $a(m) = b(m)$. [3 marks]
b. What is the cost of each plan at that time? [1 mark]
c. Find $a^{-1}(x)$ and use it to find how many months you can pay for with £111 on plan A. [3 marks]
d. Which plan is cheaper over one year? [2 marks]

Question 15

Question 15 – Composite functions with algebra (Higher) *(Section C · Problem solving)*

$f(x) = 3x - 1$ and $g(x) = x^2 + 2$.

a. Find $fg(x)$ and $gf(x)$. Expand and simplify. [4 marks]
b. Solve $fg(x) = 53$. [3 marks]
c. Show that the equation $fg(x) = gf(x)$ simplifies to $3x^2 - 3x - 1 = 0$. [3 marks]

d. Solve it to 2 decimal places. [3 marks]

Question 16

Question 16 – Inverse of an algebraic fraction (Higher) *(Section C · Problem solving)*

$f(x) = \dfrac{2x+1}{x-3}$ for $x \neq 3$.

a. Work out $f(4)$. [1 mark]
b. Show that $f^{-1}(x) = \dfrac{3x+1}{x-2}$. [5 marks]
c. Check that $f^{-1}(9) = 4$. [2 marks]
d. State a value of x that must be excluded from the domain of f^{-1}. [1 mark]

Question 17

Question 17 – Transforming a quadratic (Higher) *(Section C · Problem solving)*

$f(x) = x^2$. The graph of $y = g(x)$ is the graph of $y = f(x+2) - 5$.

a. Write $g(x)$ in the form $(x+a)^2 + b$ and state the turning point. [3 marks]
b. Describe the transformation that maps $y = f(x)$ to $y = g(x)$. [2 marks]
c. Find the points where the graph of g crosses the x-axis, to 2 decimal places. [4 marks]
d. Write the range of g. [1 mark]

Question 18

Question 18 – Finding an unknown function (Higher) *(Section C · Problem solving)*

$f(x) = x + 5$ and $fg(x) = 3x + 2$.

a. Find $g(x)$. [3 marks]
b. Find $gf(x)$. [2 marks]
c. Find $g^{-1}(x)$. [2 marks]
d. Solve $g(x) = g^{-1}(x)$. [3 marks]

Question 19

Question 19 – Transforming a graph with key points (Higher) *(Section C · Problem solving)*

The graph of $y = f(x)$ passes through $(0, 0)$, $(2, 4)$ and $(4, 0)$.

Equation	Point 1	Point 2	Point 3
$y = f(x) + 2$			
$y = f(x - 1)$			
$y = -f(x)$			
$y = f(2x)$			
$y = 2f(x)$			

a. Complete the table. [5 marks]
b. Which of these graphs has the same maximum height as $y = f(x)$? [1 mark]
c. Which graph has a maximum point at $(1, 4)$? [1 mark]

Question 20

Question 20 – Challenge: a linear function (Higher) *(Section D · Challenge)*

$f(x) = ax + b$. $f(1) = 5$ and $ff(1) = 17$.

a. Use $f(1) = 5$ to find the value of $f(5)$ and write two equations for a and b. [3 marks]
b. Solve them to find a and b. [3 marks]
c. Find $f^{-1}(x)$. [2 marks]
d. Solve $f(x) = f^{-1}(x)$. [3 marks]