Test answers - Equation of a circle and tangents

Year: Year 11 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. $(x+2)^2+(y-5)^2=9$ (1 for each bracket pair, 1 for 9; award 2 marks for fully correct) Use $(x-a)^2+(y-b)^2=r^2$ with $a=-2$, $b=5$ and $r^2=9$. Subtracting -2 gives $x+2$.

b. $(1+2)^2+(5-5)^2=9+0=9$ (1) so it is on the circle (1) Substitute $x=1$ and $y=5$: the left-hand side equals $r^2=9$.

A common error in (a) is to write $(x-2)^2$ or to leave the radius as 3 instead of $3^2=9$. In (b), the point $(1,5)$ is 3 units to the right of the centre on a horizontal line, so it is exactly one radius away.

Answer to Question 2 (4 marks)

a. Gradient of $OA=\dfrac{3}{4}$ (1); tangent gradient $-\dfrac{4}{3}$ (1) The gradient from $O(0,0)$ to $A(4,3)$ is $\frac{3}{4}$. The tangent is perpendicular to the radius, so its gradient is the negative reciprocal.

b. $y-3=-\dfrac{4}{3}(x-4)$ (1), so $3y-9=-4x+16$ and $4x+3y=25$ (1) Multiply by 3 to clear the fraction and rearrange. Check: $4\times4+3\times3=25$.

In (b), the tangent must pass through $A(4,3)$; substitution confirms this. A common error is to use the gradient of the radius, $\frac{3}{4}$, as the tangent gradient.

Answer to Question 3 (4 marks)

a. $x^2+(x+1)^2=13$ (1), $2x^2+2x+1=13$, so $x^2+x-6=0$ (1) Substitute $y=x+1$ into the circle equation and expand $(x+1)^2=x^2+2x+1$. Subtract 13 and divide by 2.

b. $(x+3)(x-2)=0$ (1), so $(2,3)$ and $(-3,-2)$ (1) Find y from $y=x+1$. Check: $4+9=13$ and $9+4=13$.

In (b), each x-value must be paired with its own y-value: $x=2$ gives $y=3$ and $x=-3$ gives $y=-2$. Since the quadratic has two different roots, the line cuts the circle twice; a repeated root would mean the line is a tangent.

Answer to Question 4 (4 marks)

a. $OA \perp TA$ (1), $TA^2 = 10^2 - 6^2 = 64$ (1), $TA =$ **8** (1) The radius $OA = 6$ is perpendicular to the tangent, so triangle OAT has a right angle at A and $OT = 10$ is the hypotenuse.

b. $\sin(ATO) = \dfrac{6}{10}$, so $36.9°$ (1) OA is opposite angle ATO and OT is the hypotenuse, so we use sine.

The point A does not need to be found to answer (a). Check (b) with cosine: $\cos(ATO) = \frac{8}{10}$ also gives $36.9°$, and the triangle has sides 6, 8 and 10.

Answer to Question 5 (4 marks)

a. Centre $(3, -1)$: gradient $\dfrac{5-(-1)}{5-3} = 3$ (1) Use the centre $(3, -1)$ and the point $P(5, 5)$: the change in y is 6 and the change in x is 2.

b. Tangent gradient $-\dfrac{1}{3}$ (1); $y - 5 = -\dfrac{1}{3}(x - 5)$ (1); $x + 3y = 20$ (1) The tangent is perpendicular to the radius. Multiply by 3 and rearrange. Check: $5 + 15 = 20$.

In (a), a common error is to use the origin as the centre; the centre comes from the brackets, $(3, -1)$. Check (b): the point $P(5, 5)$ satisfies $x + 3y = 20$.