Worksheet - Equation of a circle and tangents

Year: Year 11 · 20

Show your working.

Question 1

Question 1 – Writing the equation of a circle (Higher) *(Section A · Skills practice)*

The equation of a circle with centre (a, b) and radius r is $(x-a)^2+(y-b)^2=r^2$. Write the equation of each circle.

a. Centre $(0, 0)$, radius 5. [1 mark]
b. Centre $(2, 3)$, radius 4. [2 marks]
c. Centre $(-1, 4)$, radius 3. [2 marks]
d. Centre $(0, 0)$, radius $\sqrt{20}$. [1 mark]
e. Centre $(3, -5)$, passing through the point $(6, -1)$. [3 marks]

Question 2

Question 2 – Reading centre and radius (Higher) *(Section A · Skills practice)*

Write down the centre and the radius of each circle. Give radii as surds in simplest form when they are not whole numbers.

a. $x^2+y^2=49$ [2 marks]
b. $(x-3)^2+(y+2)^2=25$ [2 marks]
c. $(x+4)^2+y^2=12$ [2 marks]
d. $(x-1)^2+(y-1)^2=18$ [2 marks]

Question 3

Question 3 – Points on, inside and outside (Higher) *(Section A · Skills practice)*

A circle has equation $x^2+y^2=25$.

Point	$(3, 4)$	$(5, 1)$	$(2, -3)$	$(-4, -3)$	$(4, 4)$
Position					

a. Work out x^2+y^2 for each point. [3 marks]
b. State whether each point lies on, inside or outside the circle. [2 marks]
c. Which point is closest to the centre? [1 mark]

Question 4

Question 4 – The tangent at a point (Higher) *(Section A · Skills practice)*

The circle $x^2 + y^2 = 25$ passes through $A(3, 4)$. The tangent at A is perpendicular to the radius OA.

a. Find the gradient of the radius OA. [1 mark]
b. Write down the gradient of the tangent at A. [1 mark]
c. Find the equation of the tangent in the form $ax + by = c$. [3 marks]

Question 5

Question 5 – A tangent in another quadrant (Higher) *(Section A · Skills practice)*

The point $P(-6, 8)$ lies on the circle $x^2 + y^2 = 100$.

a. Show that P is on the circle. [1 mark]
b. Find the gradient of the tangent at P. [2 marks]
c. Find the equation of the tangent in the form $y = mx + c$. [3 marks]

Question 6

Question 6 – Tangents at the axes (Higher) *(Section A · Skills practice)*

The circle $x^2 + y^2 = 25$ crosses the axes at four points.

a. Write down the four points. [2 marks]
b. Write the equation of the tangent at $(5, 0)$ and explain why it is vertical. [2 marks]
c. Write the equation of the tangent at $(0, -5)$. [2 marks]
d. The four tangents form a shape. What is it and what is its area? [2 marks]

Question 7

Question 7 – The big wheel (Higher) *(Section B · Real-life practice)*

A big wheel has a radius of 10 m and its centre is 10 m above the ground. Take the ground as the line $y = 0$ and the centre as $C(0, 10)$.

a. Write the equation of the circle traced out by the passenger pods. [2 marks]
b. A pod is 6 m to the right of the centre line. Find the two possible heights. [4 marks]
c. Show that the pod at $(6, 2)$ is on the wheel. [2 marks]
d. What is the lowest point of the wheel and how far is it from the ground? [2 marks]

Question 8

Question 8 – Radar range (Higher) *(Section B · Real-life practice)*

A radar at the origin O detects ships within a distance of 50 km. All distances are in kilometres.

a. Write the equation of the boundary of the radar range. [1 mark]
b. Is a ship at $(30, 40)$ on the boundary, inside or outside? [2 marks]
c. Is a ship at $(35, 40)$ detected? How far is it beyond the edge of the range, to 2 decimal places? [3 marks]
d. Is a ship at $(20, -30)$ detected? [2 marks]

Question 9

Question 9 – Sprinkler on the lawn (Higher) *(Section B · Real-life practice)*

A sprinkler at the point $(3, -2)$ waters a circular area of radius 6 m. The lawn is on a coordinate grid in metres.

a. Write the equation of the boundary of the watered area. [2 marks]
b. Show that $(3, 4)$ is on the boundary. [2 marks]
c. Find the other boundary point on the line $y = -2$ on the right of the sprinkler. [2 marks]
d. A flowerbed is at $(6, 2)$ and another at $(8, 3)$. Which one is watered? [3 marks]

Question 10

Question 10 – A running track (Higher) *(Section B · Real-life practice)*

A circular running track has centre O at the origin and radius 40 m. A runner at $A(24, 32)$ loses control and runs along the tangent.

a. Show that A is on the track. [2 marks]
b. Find the equation of the tangent at A in the form $3x + 4y = c$. [4 marks]
c. Where does the runner cross the x-axis and the y-axis? [3 marks]
d. How far does the runner travel from A to the y-axis? Give your answer to 1 decimal place. [2 marks]

Question 11

Question 11 – Where a line meets a circle (Higher) *(Section B · Real-life practice)*

A straight path with equation $y = x + 1$ crosses a circular pond with equation $x^2 + y^2 = 25$.

a. Show that the x-coordinates of the points where the path meets the pond satisfy $x^2 + x - 12 = 0$. [3 marks]
b. Solve the equation and find the two points. [3 marks]
c. Work out the length of the path inside the pond to 2 decimal places. [3 marks]

Question 12

Question 12 – A line that just touches (Higher) *(Section B · Real-life practice)*

The circle $x^2 + y^2 = 8$ and the line $y = x + 4$ are drawn on the same axes.

a. Show that the line meets the circle where $x^2 + 4x + 4 = 0$. [3 marks]
b. Solve this equation and explain why this shows that the line is a tangent. [3 marks]
c. Find the point of contact. [1 mark]
d. Check that the radius to the point of contact is perpendicular to the line. [3 marks]

Question 13

Question 13 – Tangent length from a point (Higher) *(Section B · Real-life practice)*

A circular lake has equation $x^2 + y^2 = 25$ (units in metres). A hiker stands at $T(13, 0)$ and walks along a straight line to a point A on the lake so that TA is a tangent.

a. Explain why triangle OAT has a right angle at A. [1 mark]
b. Work out the length TA. [3 marks]
c. Work out angle ATO to 1 decimal place. [2 marks]
d. Work out the area of kite $OATB$ where TB is the other tangent. [2 marks]

Question 14

Question 14 – The circular pond (Higher) *(Section B · Real-life practice)*

A circular pond has three markers on its edge at $O(0, 0)$, $A(8, 0)$ and $B(0, 6)$, with distances in metres.

a. Explain why AB is a diameter of the pond. [2 marks]
b. Find the centre and the radius of the pond. [3 marks]
c. Write the equation of the edge of the pond. [2 marks]
d. Show that the point $(4, 8)$ is on the edge and work out the area of the pond to 1 decimal place. [3 marks]

Question 15

Question 15 – Tangent to a circle with a shifted centre (Higher) *(Section C · Problem solving)*

A circle has equation $(x - 1)^2 + (y - 2)^2 = 20$. The point $P(5, 4)$ lies on the circle.

a. Show that P is on the circle. [1 mark]
b. Find the gradient of the radius from the centre $C(1, 2)$ to P. [2 marks]
c. Find the equation of the tangent at P. [3 marks]

d. The tangent meets the axes at A and B. Work out the area of triangle OAB. [4 marks]

Question 16

Question 16 – A chord of a circle (Higher) *(Section C · Problem solving)*

The circle $x^2 + y^2 = 40$ meets the line $y = -x + 8$ at A and B.

a. Show that the points $A(6, 2)$ and $B(2, 6)$ are on both the line and the circle. [4 marks]
b. Work out the length of the chord AB to 2 decimal places. [2 marks]
c. Find the midpoint M of AB and show that OM is perpendicular to AB. [3 marks]
d. Work out the distance OM and verify with Pythagoras. [3 marks]

Question 17

Question 17 – Circle with a given diameter (Higher) *(Section C · Problem solving)*

$A(-2, 3)$ and $B(6, 9)$ are the ends of a diameter of a circle.

a. Find the centre of the circle. [2 marks]
b. Work out the radius. [3 marks]
c. Write the equation of the circle. [2 marks]
d. Show that the point $(2, 11)$ lies on the circle. [2 marks]

Question 18

Question 18 – Tangents from an outside point (Higher) *(Section C · Problem solving)*

The circle $x^2 + y^2 = 25$ has two tangents from the point $P(0, 10)$, touching the circle at A and B.

a. Work out the length of each tangent. [3 marks]
b. Work out angle OPA. [2 marks]
c. Explain why angle $APB = 60°$. [2 marks]
d. Work out the area of the quadrilateral $OAPB$ to 2 decimal places. [3 marks]

Question 19

Question 19 – Finding an unknown value for a tangent (Higher) *(Section C · Problem solving)*

The line $y = 2x + k$ is a tangent to the circle $x^2 + y^2 = 5$.

a. Show that the line meets the circle where $5x^2 + 4kx + k^2 - 5 = 0$. [3 marks]
b. For a tangent the equation has exactly one solution, so the discriminant is zero.

Show that $k^2 = 25$. [4 marks]

c. Find the two possible values of k and the points of contact when $k = 5$. [3 marks]

Question 20

Question 20 – Challenge: a circle through the origin (Higher) *(Section D · Challenge)*

A circle has equation $x^2 + y^2 - 6x - 8y = 0$.

a. Complete the square to show that the centre is $(3, 4)$ and the radius is 5. [4 marks]

b. Show that the origin is on the circle and find the equation of the tangent there. [4 marks]

c. The circle crosses the axes again at A and B. Find them and show that AB is a diameter. [4 marks]