Answers - Constructions, loci, bearings, plans and elevations

Year: Year 11

Answer to Question 1

a. East $= 090^\circ$, south $= 180^\circ$, west $= 270^\circ$, south-east $= 135^\circ$.
b. If the bearing is less than 180°, add 180°: $065 + 180 = 245^\circ$.
c. $230 - 180 = 050^\circ$; $312 - 180 = 132^\circ$; $148 + 180 = 328^\circ$.
d. Add 180° if the bearing is less than 180°, and subtract 180° if it is more. The answer must be between 000° and 360°.

Answer to Question 2

a. $4 \times 25\,000 = 100\,000$ cm $= 1000$ m $=$ **1 km**.
b. 3.5 km $= 350\,000$ cm. $350\,000 \div 25\,000 =$ **14 cm**.
c. $12 \times 50\,000 = 600\,000$ cm $=$ **6 km**.
d. 1 cm represents $25\,000$ cm $=$ **250 m**.

Answer to Question 3

a. Open the compasses to more than 4 cm. Draw an arc from A above and below the line. Keep the same radius and draw arcs from B to cross them. Join the two crossing points with a ruler. Leave the arcs showing.
b. P is 4 cm along AB from A (to the foot of the perpendicular) and 3 cm up. $AP^2 = 4^2 + 3^2 = 25$, so $AP =$ **5 cm**.
c. Place the compass point at the vertex and draw an arc crossing both arms. From each crossing point draw arcs of equal radius to meet inside the angle. Join the vertex to this meeting point. Each half is $70 \div 2 = 35^\circ$.

Answer to Question 4

a. A **circle** with centre P and radius 3 cm.
b. The **perpendicular bisector** of AB.
c. The **angle bisectors** of the angles between the lines (two lines at right angles to each other).
d. Two parallel lines 2 cm either side of AB, joined by a semicircle of radius 2 cm at each end (a "racetrack" shape). Perimeter $= 2 \times 6 + 2\pi \times 2 = 12 + 4\pi =$ **24.6 cm**.

Answer to Question 5

a. A **square** of side 6 cm with its diagonals drawn (the apex is above the centre).
b. An **isosceles triangle** with base 6 cm and height 4 cm.
c. Half the base is 3 cm. Slant height $= \sqrt{3^2 + 4^2} = \sqrt{25} =$ **5 cm**.
d. One face $= \frac{1}{2} \times 6 \times 5 =$ **15 cm²**. Four faces $= 4 \times 15 =$ **60 cm²**.

Answer to Question 6

a. $130 - 40 = 90$, so the turn between the two legs is 90° (the legs are perpendicular).
b. $\sqrt{12^2 + 9^2} = \sqrt{225} =$ **15 km**.
c. The angle at the start between the first leg and the line to the ship is $\tan^{-1}\left(\frac{9}{12}\right) = 36.9^\circ$. The bearing is $40 + 36.9 = 76.9 \approx 077^\circ$.
d. Back bearing $= 77 + 180 = 257^\circ$.

Answer to Question 7

a. At A the bearing back to P is $060 + 180 = 240^\circ$ and the ferry leaves on 150°. The angle between these is $240 - 150 = 90^\circ$.
b. $PB = \sqrt{20^2 + 20^2} = \sqrt{800} =$ **28.28 km**.
c. Triangle PAB is isosceles and right-angled, so angle $APB = 45^\circ$. The bearing is $060 + 45 = 105^\circ$.
d. $105 + 180 = 285^\circ$.

Answer to Question 8

a. $120 + 180 = 300^\circ$.
b. The bearing of H from L is 300° and the bearing of S from L is 210°. $300 - 210 = 90^\circ$.
c. $HS = \sqrt{15^2 + 8^2} = \sqrt{289} =$ **17 km**.
d. Angle $LHS = \tan^{-1}\left(\frac{8}{15}\right) = 28.1^\circ$. The ship is further round clockwise than L: $120 + 28.1 = 148.1^\circ$.

Answer to Question 9

a. 1 cm represents 0.5 km: $4.2 \times 0.5 =$ **2.1 km**, $6.8 \times 0.5 =$ **3.4 km**, $3.0 \times 0.5 =$ **1.5**

km.
b. $2.1 + 3.4 + 1.5 =$ **7.0 km**.
c. $7 \div 4 = 1.75$ hours = **1 hour 45 minutes**.
d. Straight line: $9.6 \times 0.5 = 4.8$ km, so it is $7.0 - 4.8 =$ **2.2 km** shorter.

Answer to Question 10

a. Draw the perpendicular bisector of AB, which is 5 m from A. The region is on the A side of this line.
b. A circle of radius 6 cm centred on A (inside the circle).
c. No. The point is 6 m from A along AB, which is beyond the bisector (5 m from A), so it is nearer to B (only 4 m away).
d. Least distance: just above **0 m** (very near A). Greatest: **5 m** (the bisector), since 5 is less than 6.

Answer to Question 11

a. A corner is 10 m across and 6 m up from the centre: $\sqrt{10^2 + 6^2} = \sqrt{136} =$ **11.66 m**.
b. $11.66 < 15$, so **yes**, the whole office is covered.
c. The diagonal is $\sqrt{20^2 + 12^2} = \sqrt{544} = 23.3$ m. This is more than 15 m, so **no**.
d. $\pi \times 15^2 =$ **706.9 m²**.

Answer to Question 12

a. $\sqrt{200^2 + 150^2} = \sqrt{62\,500} =$ **250 km**.
b. The angle east of north is $\tan^{-1}\left(\frac{150}{200}\right) = 36.9°$, so the bearing is $037°$.
c. Back bearing $= 37 + 180 = 217°$ (using the exact angle, $216.9°$).
d. $250 \div 250 =$ **1 hour**.

Answer to Question 13

a. $6 + 7 = 13 > 10$, so the circles overlap.
b. It is 4 km from A (< 6) and $10 - 4 = 6$ km from B (< 7). **Yes**, it receives both.
c. Signal A reaches up to 6 km from A. Signal B reaches back to $10 - 7 = 3$ km from A. Both signals are received between 3 km and 6 km: a stretch of **3 km**.
d. It is 8 km from A (> 6) so no signal from A; it is $10 - 8 = 2$ km from B (< 7), so it receives only signal B.

Answer to Question 14

a. 8.5 m $= 850$ cm, so **8.5 cm** on the plan; 6 m $=$ **6 cm** on the plan.
b. $8.5 \times 6 =$ **51 m²**.
c. $51 \times 4.20 =$ **£214.20**.
d. Perimeter $= 2 \times (8.5 + 6) = 29$ m. Cost $= 29 \times 18 =$ **£522**.

Answer to Question 15

a. The bearing of A from B is $050 + 180 = 230°$. The bearing of C from B is $140°$. The angle between these is $230 - 140 = 90°$.
b. $AC = \sqrt{24^2 + 18^2} = \sqrt{900} =$ **30 km**.
c. Angle $BAC = \tan^{-1}\left(\frac{18}{24}\right) = 36.9°$. Bearing $= 50 + 36.9 = 86.9 \approx 087°$.
d. $86.9 + 180 = 266.9 \approx 267°$.

Answer to Question 16

a. The perpendicular bisector of AB is the line $x = 5$. The region is the side containing A ($x < 5$).
b. A circle of radius 4 cm centred at D; the region is inside it.
c. $x = 3 < 5$, and the distance from D is $\sqrt{3^2 + 1^2} = \sqrt{10} = 3.16 < 4$. **Yes**.
d. $(4, 1)$: $x < 5$ but the distance from D is $\sqrt{16 + 25} = 6.4 > 4$, so **no**. $(6, 5)$: $x > 5$, so it is nearer to B, so **no**.

Answer to Question 17

a. The plan is the L-shape: 9 cm long and 3 cm wide, with a 3 cm by 3 cm square sticking up at one end. The front elevation is a rectangle 9 cm wide and 5 cm high (the L-shape from the front shows a 9 by 5 rectangle).
b. Area of L $= 4 \times 9 = 36$ cm². Volume $= 36 \times 5 =$ **180 cm³**.
c. Top and bottom: $2 \times 36 = 72$ cm². The perimeter of the L is 10 edges of 3 cm $= 30$ cm, so the vertical sides give $30 \times 5 = 150$ cm². Total $= 72 + 150 =$ **222 cm²**.

Answer to Question 18

a. Draw $AB = 7$ cm. At A use a protractor to draw a ray at $50°$ to AB. Mark C on the ray 6 cm from A with compasses. Join B to C.
b. $BC^2 = 7^2 + 6^2 - 2 \times 7 \times 6 \cos 50° = 85 - 84 \times 0.6428 = 85 - 54.0 = 31.0$, so

$BC =$ **5.57 cm**.

c. Yes: 5.6 cm agrees with 5.57 cm to 1 d.p.

d. The bisector makes 25° with AB.

Answer to Question 19

a. At the corner the shed takes up a 90° angle, so the goat has the other 270°, which is three quarters of a full turn.

b. $\frac{3}{4} \times \pi \times 5^2 = \frac{3}{4} \times 78.54 =$ **58.9 m²**.

c. The rope is 5 m but the shorter side is 6 m, so the rope is too short to reach the next corner on either side.

Answer to Question 20

a. At the corner the goat has 270° so $\frac{3}{4}$ of a circle of radius 10.

b. Going round the 8 m side, 8 m of rope is used to reach the next corner, leaving $10 - 8 = 2$ m: a quarter circle of radius 2. Going round the 6 m side leaves $10 - 6 = 4$ m: a quarter circle of radius 4.

c. Area $= \frac{3}{4}\pi 10^2 + \frac{1}{4}\pi 2^2 + \frac{1}{4}\pi 4^2 = 75\pi + \pi + 4\pi = 80\pi =$ **251.3 m²**.