Test answers - Conditional probability

Year: Year 11 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. $\frac{26}{48} = \frac{13}{24}$ (1 for 26, 1 for 48) The condition 'likes' means we only count the $22 + 26 = 48$ students who like the menu, and 26 of them are girls.

b. $\frac{22}{40} = \frac{11}{20}$ (1 for 22, 1 for 40) The condition 'boy' means we only count the $22 + 18 = 40$ boys, and 22 of them like the menu.

A common error in (a) is to divide by 80, the whole group, which gives $P(\text{girl and likes}) = \frac{26}{80}$ instead.

Answer to Question 2 (4 marks)

a. GG: $\frac{4}{10} \times \frac{3}{9} = \frac{12}{90}$; YY: $\frac{6}{10} \times \frac{5}{9} = \frac{30}{90}$ (1 each); total $\frac{42}{90} = \frac{7}{15}$ (1) 'Same colour' means both green or both yellow. The beads are not replaced, so the second fraction has 9 as the denominator. Add the two products.

b. $\frac{4}{9}$ (1) After a yellow bead is taken, 4 green beads remain out of 9.

Check (a): the probability of different colours is $2 \times \frac{4}{10} \times \frac{6}{9} = \frac{48}{90}$, and $\frac{42}{90} + \frac{48}{90} = 1$.

Answer to Question 3 (4 marks)

a. $\frac{0.35}{0.5} = \mathbf{0.7}$ (1 for the formula, 1 for the answer) Divide the probability of both events by the probability of B.

b. $0.6 \times 0.5 = 0.3$, which is not 0.35, so **not independent** (1 for the product, 1 for the conclusion) For independent events $P(A \cap B)$ would equal $P(A) \times P(B)$.

Another way to answer (b): $P(A \mid B) = 0.7$, which is different from $P(A) = 0.6$, so knowing that B happened changes the probability of A. This also shows that the events are not independent.

Answer to Question 4 (4 marks)

a. $0.25 \times 0.3 + 0.75 \times 0.1 = 0.075 + 0.075 = \mathbf{0.15}$ (1 for each product) The bus can be late in two ways: rain and late, or no rain and late. The probability of no rain is $1 - 0.25 = 0.75$.

b. $\frac{0.075}{0.15} = \mathbf{0.5}$ (1 for the numerator, 1 for the answer) Divide the probability of 'rain and late' by the total probability of being late. So when the bus is late, rain and no rain are equally likely.

Answer to Question 5 (4 marks)

a. $\frac{4}{4+n} \times \frac{3}{3+n} = \frac{1}{6}$ (1), so $72 = (4+n)(3+n) = n^2 + 7n + 12$, giving $n^2 + 7n - 60 = 0$ (1) There are $4+n$ counters in total, and after one red is taken there are 3 red counters out of $3+n$. Cross-multiply and expand $(4+n)(3+n)$.

b. $(n+12)(n-5) = 0$ (1), so $n = 5$ (1) We need two numbers with product -60 and sum 7: 12 and -5. A number of counters cannot be negative, so $n = 5$. Check: $\frac{4}{9} \times \frac{3}{8} = \frac{12}{72} = \frac{1}{6}$.