# Worksheet - Conditional probability

Year: Year 11 · 20

Show your working.

## Question 1

**Question 1 – A two-way table of journeys (Higher)** *(Section A · Skills practice)*

A survey of 60 pupils recorded how they travel to school.

| | Walk | Bus |
|---|---|---|
| Boys | 12 | 18 |
| Girls | 15 | 15 |

a. A pupil is chosen at random. Find the probability that the pupil goes by bus. [2 marks]
b. Find the probability that the pupil is a boy who goes by bus. [1 mark]
c. A girl is chosen at random. Find the probability that she walks. [2 marks]
d. A pupil who walks is chosen at random. Find the probability that the pupil is a girl. [2 marks]

## Question 2

**Question 2 – A Venn diagram of drinks (Higher)** *(Section A · Skills practice)*

40 people were asked if they drink tea, coffee or both. 14 drink only tea, 9 drink only coffee, 6 drink both and 11 drink neither.

a. Draw a Venn diagram (or write the four numbers). [2 marks]
b. Find $P(\text{neither})$. [1 mark]
c. Find $P(\text{coffee} \mid \text{tea})$. [2 marks]
d. Find $P(\text{tea} \mid \text{coffee})$. [2 marks]

## Question 3

**Question 3 – Counters without replacement (Higher)** *(Section A · Skills practice)*

A bag contains 5 red counters and 3 blue counters. Two counters are taken at random without replacement.

a. Complete the probability tree: first draw red $\frac{5}{8}$, blue $\frac{3}{8}$; second draw after red: red $\frac{4}{7}$, blue $\frac{3}{7}$; after blue: red $\frac{5}{7}$, blue $\frac{2}{7}$. [3 marks]
b. Find the probability that both counters are red. [2 marks]
c. Find the probability that both counters are blue. [2 marks]

d. Find the probability that the counters are different colours. [3 marks]

## Question 4

**Question 4 – Conditional probability from the tree (Higher)** *(Section A · Skills practice)*

Use the counters in Question 3.

a. Given that the first counter is red, find the probability that the second is red. [1 mark]
b. Find the probability that at least one counter is blue. [2 marks]
c. Given that at least one counter is blue, find the probability that both are blue. [3 marks]

## Question 5

**Question 5 – The conditional probability formula (Higher)** *(Section A · Skills practice)*

$P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$.

a. Use the formula $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ to find $P(A \mid B)$. [2 marks]
b. Find $P(B \mid A)$. [2 marks]
c. Are $A$ and $B$ independent? Explain. [2 marks]

## Question 6

**Question 6 – Two forms of the multiplication rule (Higher)** *(Section A · Skills practice)*

For two events $A$ and $B$: $P(A \cap B) = P(A) \times P(B \mid A)$.

a. $P(A) = 0.6$ and $P(B \mid A) = 0.25$. Find $P(A \cap B)$. [2 marks]
b. $P(A \cap B) = 0.12$ and $P(B) = 0.3$. Find $P(A \mid B)$. [2 marks]
c. $P(A \cap B) = 0.18$ and $P(A) = 0.45$. Find $P(B \mid A)$. [2 marks]

## Question 7

**Question 7 – Rain and being late (Higher)** *(Section B · Real-life practice)*

The probability that it rains on a school morning is 0.3. If it rains, the probability that Maya is late is 0.4. If it does not rain, the probability that Maya is late is 0.1.

a. Draw a tree diagram and label all the branches. [3 marks]
b. Find the probability that it rains and Maya is late. [2 marks]
c. Find the probability that Maya is late. [2 marks]
d. Maya is late. Find the probability that it was raining. [3 marks]

## Question 8

**Question 8 – A medical test (Higher)** *(Section B · Real-life practice)*

2% of a population have a disease. A test gives a positive result for 95% of people who have the disease and for 4% of people who do not.

a. Draw a tree diagram. [3 marks]
b. Work out the probability that a randomly chosen person tests positive. [3 marks]
c. A person tests positive. Work out the probability that they actually have the disease, to 3 decimal places. [3 marks]
d. Explain why this probability is lower than many people expect. [1 mark]

## Question 9

**Question 9 – Mints and toffees (Higher)** *(Section B · Real-life practice)*

A box has 12 sweets: 7 mints and 5 toffees. Ali takes two sweets at random, one after the other, and eats them.

a. Find the probability that both are mints. [2 marks]
b. Find the probability that they are different. [3 marks]
c. Given that the first is a mint, find the probability that the second is a toffee. [1 mark]
d. Given that the sweets are different, find the probability that the first was a mint. [3 marks]

## Question 10

**Question 10 – School clubs (Higher)** *(Section B · Real-life practice)*

100 pupils were asked about school clubs. 35 go to drama, 40 go to music and 15 go to both.

a. Draw a Venn diagram showing the numbers in each region. [3 marks]
b. Find $P(\text{neither club})$. [1 mark]
c. Find $P(\text{music} \mid \text{drama})$. [2 marks]
d. Find $P(\text{drama} \mid \text{music})$. [2 marks]
e. Are going to drama and going to music independent? [2 marks]

## Question 11

**Question 11 – A football team (Higher)** *(Section B · Real-life practice)*

A team scores first in 55% of its matches. When it scores first, it wins 60% of the time. When it does not score first, it wins 20% of the time.

a. Draw a tree diagram. [3 marks]
b. Find the probability that the team wins a match. [3 marks]

c. The team won. Find the probability that it scored first. [3 marks]
d. Find the expected number of wins in 40 matches. [1 mark]

## Question 12

**Question 12 – Numbered cards (Higher)** *(Section B · Real-life practice)*

Ten cards are numbered 1 to 10. Two cards are taken at random without replacement.

a. Find the probability that both numbers are even. [2 marks]
b. Find the probability that the sum of the two numbers is even. [3 marks]
c. Given that the sum is even, find the probability that both numbers are odd. [2 marks]

## Question 13

**Question 13 – Two traffic lights (Higher)** *(Section B · Real-life practice)*

A driver meets two sets of traffic lights. The probability that the first is green is 0.6. If the first is green, the probability that the second is green is 0.7. If the first is red, the probability that the second is green is 0.3.

a. Find the probability that both lights are green. [2 marks]
b. Find the probability that the second light is green. [3 marks]
c. Given that the second light is red, find the probability that the first light was green. [3 marks]

## Question 14

**Question 14 – Pets in Years 10 and 11 (Higher)** *(Section B · Real-life practice)*

200 pupils were asked whether they own a pet.

| | Has a pet | No pet |
|---|---|---|
| Year 10 | 60 | 40 |
| Year 11 | 45 | 55 |

a. Find the probability that a random pupil has a pet. [2 marks]
b. Find $P(\text{Year 11} \mid \text{has a pet})$. [2 marks]
c. Find $P(\text{has a pet} \mid \text{Year 10})$. [2 marks]
d. Are owning a pet and being in Year 10 independent? [2 marks]

## Question 15

**Question 15 – Choosing a bag with a coin (Higher)** *(Section C · Problem solving)*

Bag A has 3 red and 2 blue counters. Bag B has 1 red and 4 blue counters. A fair coin is tossed. If it shows heads, a counter is taken from Bag A; if tails, from Bag B.

a. Draw a tree diagram. [3 marks]
b. Find the probability that the counter is red. [3 marks]
c. The counter is red. Find the probability that it came from Bag A. [3 marks]
d. The experiment is repeated 200 times. Estimate how many red counters will be taken. [1 mark]

## Question 16

**Question 16 – Three draws without replacement (Higher)** *(Section C · Problem solving)*

A box holds 4 white and 6 black discs. Three discs are taken at random without replacement.

a. Show that the probability that all three are black is $\frac{1}{6}$. [3 marks]
b. Find the probability that at least one is white. [1 mark]
c. Find the probability that exactly one is white. [4 marks]
d. Find the probability that all three are white. [2 marks]

## Question 17

**Question 17 – Independent events and an unknown probability (Higher)** *(Section C · Problem solving)*

$A$ and $B$ are independent events. $P(A) = 0.4$ and $P(A \cup B) = 0.7$.

a. Write down an expression for $P(A \cup B)$ in terms of $P(B) = x$. [2 marks]
b. Solve to find $x$. [3 marks]
c. Find $P(A \cap B)$ and $P(A \mid B)$. [2 marks]
d. Find the probability that exactly one of $A$ and $B$ happens. [2 marks]

## Question 18

**Question 18 – Working backwards to find the number of sweets** *(Section C · Problem solving)*

A bag has 10 sweets, of which $n$ are orange. Two sweets are taken at random without replacement. The probability that both are orange is $\frac{1}{15}$.

a. Show that $n(n-1) = 6$. [3 marks]
b. Solve the equation to find $n$. [3 marks]
c. Find the probability that exactly one sweet is orange. [3 marks]

## Question 19

**Question 19 – Quality control (Higher)** *(Section C · Problem solving)*

A factory has two machines. Machine A makes 60% of the items and 2% of its items are faulty. Machine B makes the rest and 5% of its items are faulty.

a. Draw a tree diagram. [3 marks]
b. Find the probability that an item chosen at random is faulty. [3 marks]
c. A faulty item is found. Find the probability that it was made by Machine B. [3 marks]
d. In a batch of 1 000 items, how many are expected to be faulty? [1 mark]

## Question 20

**Question 20 – Challenge: the two children** *(Section D · Challenge)*

A family has two children. Assume that each child is equally likely to be a boy or a girl, independently.

a. List the four equally likely outcomes in order of birth (older child first). [1 mark]
b. You are told that at least one of the children is a boy. Find the probability that both are boys. [3 marks]
c. You are told that the older child is a boy. Find the probability that both are boys. [2 marks]
d. Explain why the answers to (b) and (c) are different. [2 marks]