Answers - Conditional probability

Year: Year 11

Answer to Question 1

a. $\frac{18+15}{60} = \frac{33}{60} = \frac{11}{20}$. Add the boys and girls who go by bus: $18+15=33$ out of 60 pupils, then simplify by dividing by 3.

b. $\frac{18}{60} = \frac{3}{10}$. The boys who go by bus are one cell of the table: 18 out of all 60 pupils.

c. Only the 30 girls matter: $\frac{15}{30} = \frac{1}{2}$. The condition 'a girl is chosen' means we only look at the girls' row, so the denominator is 30, not 60.

d. Only the 27 pupils who walk matter: $\frac{15}{27} = \frac{5}{9}$. The condition 'walks' means we only look at the walk column: $12+15=27$ pupils, of whom 15 are girls.

Answer to Question 2

a. Tea only 14, both 6, coffee only 9, neither 11, and $14+6+9+11=40$.

b. $\frac{11}{40}$. 11 of the 40 people drink neither, so the probability is $\frac{11}{40}$.

c. There are $14+6=20$ tea drinkers and 6 of them also drink coffee: $\frac{6}{20} = \frac{3}{10}$.

d. There are $9+6=15$ coffee drinkers and 6 also drink tea: $\frac{6}{15} = \frac{2}{5}$.

Answer to Question 3

a. After a red counter is removed there are 4 red and 3 blue among 7. After a blue counter is removed there are 5 red and 2 blue among 7. The numbers on the branches are as given.

b. $\frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$.

c. $\frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$.

d. Red then blue: $\frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$. Blue then red: $\frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$. Total $= \frac{30}{56} = \frac{15}{28}$.

Answer to Question 4

a. After a red counter is taken there are 4 red and 3 blue left: $\frac{4}{7}$. The first counter is not replaced, so there are only 7 counters left for the second draw.

b. $1 - P(\text{both red}) = 1 - \frac{5}{14} = \frac{9}{14}$. 'At least one blue' is the complement of 'both red', so we subtract from 1.

c. $P(\text{both blue} \mid \text{at least one blue}) = \frac{3/28}{9/14} = \frac{3}{28} \times \frac{14}{9} = \frac{42}{252} = \frac{1}{6}$. Both blue is part of 'at least one blue', so we divide the probability of both blue by the probability of at least one blue.

Answer to Question 5

a. $P(A \mid B) = \frac{0.2}{0.4} = \mathbf{0.5}$. Divide the probability of both events by the probability of the condition B.

b. $P(B \mid A) = \frac{0.2}{0.5} = \mathbf{0.4}$. This time the condition is A, so we divide by $P(A) = 0.5$.

c. $P(A) \times P(B) = 0.5 \times 0.4 = 0.2 = P(A \cap B)$, so **yes, they are independent**. Also $P(A \mid B) = P(A)$. For independent events, knowing that B happened does not change the probability of A.

Answer to Question 6

a. $0.6 \times 0.25 = \mathbf{0.15}$. Multiply the probability of A by the probability of B given A. The answer is 0.15.

b. $\frac{0.12}{0.3} = \mathbf{0.4}$. Rearrange the formula: $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$. The answer is 0.4.

c. $\frac{0.18}{0.45} = \mathbf{0.4}$. Rearrange the formula: $P(B \mid A) = \frac{P(A \cap B)}{P(A)}$. The answer is 0.4.

Answer to Question 7

a. First branches: rain 0.3, no rain 0.7. Second branches after rain: late 0.4, on time 0.6. After no rain: late 0.1, on time 0.9. The probabilities on each pair of branches add up to 1.

b. $0.3 \times 0.4 = \mathbf{0.12}$. Multiply along the branches rain then late.

c. $0.12 + 0.7 \times 0.1 = 0.12 + 0.07 = \mathbf{0.19}$. Maya can be late in two ways: rain and late, or no rain and late. Add the two products.

d. $P(\text{rain} \mid \text{late}) = \frac{0.12}{0.19} = \frac{12}{19} \approx 0.63$. The condition is that Maya is late, so we

divide the 'rain and late' probability by the total probability of being late.

Answer to Question 8

a. First branches: disease 0.02, no disease 0.98. Then positive 0.95 or negative 0.05 after disease; positive 0.04 or negative 0.96 after no disease.
b. $0.02 \times 0.95 + 0.98 \times 0.04 = 0.019 + 0.0392 =$ **0.0582**.
c. $P(\text{disease} \mid \text{positive}) = \dfrac{0.019}{0.0582} =$ **0.326**.
d. Most people are healthy, so even a small false positive rate produces many more false positives than true positives.

Answer to Question 9

a. $\dfrac{7}{12} \times \dfrac{6}{11} = \dfrac{42}{132} = \dfrac{7}{22}$.
b. Mint then toffee: $\dfrac{7}{12} \times \dfrac{5}{11} = \dfrac{35}{132}$. Toffee then mint: $\dfrac{5}{12} \times \dfrac{7}{11} = \dfrac{35}{132}$. Total $= \dfrac{70}{132} = \dfrac{35}{66}$.
c. After taking a mint: 6 mints and 5 toffees in 11, so $\dfrac{5}{11}$.
d. By symmetry the two ways are equally likely: $\dfrac{35/132}{70/132} = \dfrac{1}{2}$.

Answer to Question 10

a. Both 15, drama only $35 - 15 = 20$, music only $40 - 15 = 25$, neither $100 - 60 = 40$.
b. $\dfrac{40}{100} =$ **0.4**.
c. $\dfrac{15}{35} = \dfrac{3}{7}$. Only the 35 drama pupils count, and 15 of them also go to music.
d. $\dfrac{15}{40} = \dfrac{3}{8}$. Only the 40 music pupils count, and 15 of them also go to drama.
e. $P(\text{drama}) \times P(\text{music}) = 0.35 \times 0.4 = 0.14$, but $P(\text{both}) = 0.15$. These are different, so the events are **not independent**.

Answer to Question 11

a. First branches: scores first 0.55, concedes first 0.45. Then win 0.6 / not win 0.4 after scoring first; win 0.2 / not win 0.8 after conceding first.

b. $0.55 \times 0.6 + 0.45 \times 0.2 = 0.33 + 0.09 = \mathbf{0.42}$.
c. $\dfrac{0.33}{0.42} = \dfrac{11}{14} \approx 0.786$.
d. $40 \times 0.42 = \mathbf{16.8}$, so about **17 wins**.

Answer to Question 12

a. There are 5 even cards: $\dfrac{5}{10} \times \dfrac{4}{9} = \dfrac{2}{9}$.
b. The sum is even when both are even or both are odd. Both odd is also $\dfrac{5}{10} \times \dfrac{4}{9} = \dfrac{2}{9}$. Total $= \dfrac{2}{9} + \dfrac{2}{9} = \dfrac{4}{9}$.
c. $\dfrac{2/9}{4/9} = \dfrac{1}{2}$. Both odd is one of the two equally likely ways to get an even sum.

Answer to Question 13

a. $0.6 \times 0.7 = \mathbf{0.42}$. Multiply along the branches green then green.
b. $0.42 + 0.4 \times 0.3 = 0.42 + 0.12 = \mathbf{0.54}$. The second light can be green in two ways, so we add the two products: green then green, and red then green.
c. $P(\text{second red}) = 1 - 0.54 = 0.46$. The first green and second red has probability $0.6 \times 0.3 = 0.18$. So $\dfrac{0.18}{0.46} = \dfrac{9}{23} \approx 0.39$. Divide the probability of 'first green and second red' by the probability of 'second red'.

Answer to Question 14

a. $\dfrac{60 + 45}{200} = \dfrac{105}{200} = \mathbf{0.525}$. Add the two Year groups in the 'has a pet' column, then divide by 200 pupils.
b. $\dfrac{45}{105} = \dfrac{3}{7}$. The condition 'has a pet' means only the 105 pupils with a pet count, and 45 of them are in Year 11.
c. $\dfrac{60}{100} = \mathbf{0.6}$. The condition 'Year 10' means only the 100 Year 10 pupils count, and 60 of them have a pet.
d. $P(\text{pet}) = 0.525$ but $P(\text{pet} \mid \text{Year 10}) = 0.6$. They are different, so the events are **not independent**. For independent events the conditional probability would equal the overall probability.

Answer to Question 15

a. First branches: A 0.5, B 0.5. After A: red $\frac{3}{5}$, blue $\frac{2}{5}$. After B: red $\frac{1}{5}$, blue $\frac{4}{5}$. The probabilities on each pair of branches add up to 1.
b. $0.5 \times 0.6 + 0.5 \times 0.2 = 0.3 + 0.1 = \mathbf{0.4}$. A red counter can come from Bag A or from Bag B, so we add the two products.
c. $\dfrac{0.3}{0.4} = \mathbf{0.75}$. Divide the probability of 'Bag A and red' by the total probability of red.
d. $200 \times 0.4 = \mathbf{80}$. Multiply the number of trials by the probability of red. We expect about 80 red counters.

Answer to Question 16

a. $\dfrac{6}{10} \times \dfrac{5}{9} \times \dfrac{4}{8} = \dfrac{120}{720} = \dfrac{1}{6}$.
b. $1 - \dfrac{1}{6} = \dfrac{5}{6}$.
c. One white can be in three positions: WBB, BWB, BBW. WBB $= \dfrac{4}{10} \times \dfrac{6}{9} \times \dfrac{5}{8} = \dfrac{120}{720}$. The other two orders have the same probability, so the total is $3 \times \dfrac{120}{720} = \dfrac{360}{720} = \dfrac{1}{2}$.
d. $\dfrac{4}{10} \times \dfrac{3}{9} \times \dfrac{2}{8} = \dfrac{24}{720} = \dfrac{1}{30}$. Multiply the probabilities of three whites, reducing the number of white discs and the total each time.

Answer to Question 17

a. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + x - 0.4x$ (independent so $P(A \cap B) = 0.4x$).
b. $0.4 + 0.6x = 0.7$, so $0.6x = 0.3$ and $x = 0.5$.
c. $P(A \cap B) = 0.4 \times 0.5 = \mathbf{0.2}$. $P(A \mid B) = \dfrac{0.2}{0.5} = \mathbf{0.4}$ (equal to $P(A)$, as expected).
d. Exactly one: $P(A \cup B) - P(A \cap B) = 0.7 - 0.2 = \mathbf{0.5}$.

Answer to Question 18

a. $\dfrac{n}{10} \times \dfrac{n-1}{9} = \dfrac{n(n-1)}{90} = \dfrac{1}{15}$, so $n(n-1) = \dfrac{90}{15} = 6$. Multiply the probabilities along the branches, cross-multiply, then simplify.
b. $n^2 - n - 6 = 0$, $(n-3)(n+2) = 0$. As n is positive, $n = 3$. Rearrange to make one side zero and factorise. A number of sweets cannot be negative.
c. Orange then other: $\dfrac{3}{10} \times \dfrac{7}{9} = \dfrac{21}{90}$. Other then orange: $\dfrac{7}{10} \times \dfrac{3}{9} = \dfrac{21}{90}$. Total

$= \frac{42}{90} = \frac{7}{15}$. Exactly one orange can happen in two orders, so we add the two products.

Answer to Question 19

a. First branches: A 0.6, B 0.4. After A: faulty 0.02. After B: faulty 0.05. The probabilities on each pair of branches add up to 1, so the 'not faulty' branches are 0.98 and 0.95.
b. $0.6 \times 0.02 + 0.4 \times 0.05 = 0.012 + 0.020 =$ **0.032**. A faulty item can come from Machine A or from Machine B, so we add the two products.
c. $\frac{0.020}{0.032} =$ **0.625** (or $\frac{5}{8}$). Divide the probability of 'Machine B and faulty' by the total probability of faulty.
d. $1000 \times 0.032 =$ **32 items**. Multiply the number of items by the probability of a fault.

Answer to Question 20

a. BB, BG, GB, GG.
b. At least one boy removes only GG, leaving BB, BG, GB. Only BB is "both boys": $\frac{1}{3}$.
c. The older child is a boy leaves BB and BG. Only BB has two boys: $\frac{1}{2}$.
d. The two pieces of information are different. "At least one boy" keeps three outcomes; "the older is a boy" keeps only two, because it also tells you which child is the boy. More specific information changes the conditional probability.