Test answers - Circle theorems

Year: Year 11 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. $ACB = 118 \div 2 = 59^\circ$, because the angle at the centre is twice the angle at the circumference (1 each) Angle ACB stands on the same arc AB as the angle at the centre, so it is half of 118°.
b. $ACB = 90^\circ$ (angle in a semicircle), so $ABC = 180 - 90 - 37 = 53^\circ$ (1 each) The angles of triangle ABC add up to 180°.

In (b), the theorem name must be stated for the reason mark.

Answer to Question 2 (4 marks)

a. $4x + 5 + 2x + 25 = 180$ (1), $6x = 150$ (1), $x = 25$ (1) Angles A and C are opposite angles of a cyclic quadrilateral, so they add up to 180°. Collect the terms: $6x + 30 = 180$, then subtract 30 and divide by 6.
b. 105° (1) Substitute $x = 25$ into $4x + 5$: $4 \times 25 + 5 = 105$. Check: angle $C = 2 \times 25 + 25 = 75^\circ$ and $105 + 75 = 180$.

Do not set the two angles equal: $4x + 5 = 2x + 25$ gives $x = 10$, which is wrong because opposite angles of a cyclic quadrilateral are supplementary, not equal.

Answer to Question 3 (4 marks)

a. Angle $OAT = 90^\circ$ (1), $TA^2 = 17^2 - 8^2 = 225$ (1), $TA =$ **15 cm** (1) The tangent TA is perpendicular to the radius OA, so triangle OAT has a right angle at A and $OT = 17$ cm is the hypotenuse. Subtract the squares, then take the square root.
b. $\sin(ATO) = \frac{8}{17}$, so $ATO = 28.1^\circ$ (1) $OA = 8$ cm is opposite angle ATO and $OT = 17$ cm is the hypotenuse, so we use sine and then the inverse sine key.

Answer to Question 4 (4 marks)

a. The angle in the alternate segment is 64° (1) and the angle at the centre is twice this (1): $AOB = 128^\circ$ (1) By the alternate segment theorem, the angle between the tangent and the chord AB equals any angle in the opposite segment.
b. $(180 - 128) \div 2 = 26^\circ$ (1) Triangle OAB is isosceles because $OA = OB$ are radii.

Check (b): $26^\circ + 26^\circ + 128^\circ = 180^\circ$. Another route: angle $OAT = 90^\circ$ because the tangent is perpendicular to the radius, so $OAB = 90 - 64 = 26^\circ$.

Answer to Question 5 (4 marks)

a. The half-chord is 15, the distance from the centre to the chord is $r - 5$ and Pythagoras gives the result (1)
b. $r^2 = 225 + r^2 - 10r + 25$ (1), $10r = 250$ (1), $r = 25$ **m** (1) The r^2 terms cancel, which leaves a linear equation in r.

In (a), draw the radius to one end of the chord: the radius is the hypotenuse, half of the 30 m chord is one side, and the perpendicular from the centre to the chord, $r - 5$, is the other. Check (b): $15^2 + 20^2 = 225 + 400 = 625 = 25^2$.