Worksheet - Circle theorems

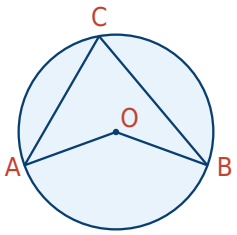
Year: Year 11 · 20

Show your working.

Question 1

Question 1 – Angle at the centre *(Section A · Skills practice)*

In each part, A, B and C are points on a circle with centre O.

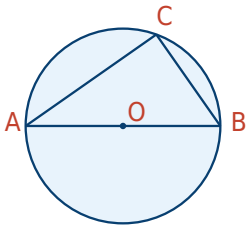


a. Angle $ACB = 38°$. Find angle AOB. Give a reason. [2 marks]
b. Angle $AOB = 130°$. Find angle ACB. Give a reason. [2 marks]
c. Angle $ACB = x$ and angle $AOB = (3x - 20)°$. Form an equation and find x. [3 marks]

Question 2

Question 2 – Angle in a semicircle *(Section A · Skills practice)*

AB is a diameter of a circle and C is a point on the circumference. Angle $BAC = 34°$.

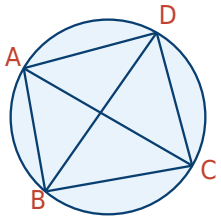


a. Write down angle ACB and give a reason. [2 marks]
b. Work out angle ABC. [2 marks]
c. M is the centre of the circle. Show that triangle AMC is isosceles and find angle AMC. [3 marks]

Question 3

Question 3 – Angles in the same segment *(Section A · Skills practice)*

A, B, C and D are on a circle. AB and CD are chords that cross. Angle $ABD = 42°$, angle $CAD = 35°$ and angle $BAC = 28°$.

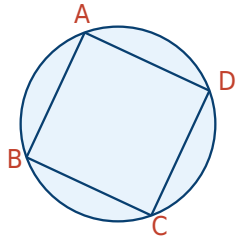


a. Find angle ACD and give a reason. [2 marks]
b. Find angle CBD. [2 marks]
c. Find angle BDC. [2 marks]

Question 4

Question 4 – Cyclic quadrilaterals *(Section A · Skills practice)*

$ABCD$ is a cyclic quadrilateral.

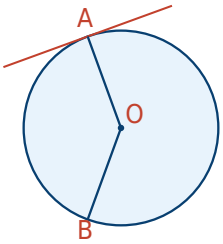


a. Angle $A = 108°$. Find angle C. [2 marks]
b. Angle $B = 85°$. Find angle D. [1 mark]
c. Angle $B = (2x + 10)°$ and angle $D = (3x - 20)°$. Find x and both angles. [4 marks]

Question 5

Question 5 – Tangents and radii *(Section A · Skills practice)*

O is the centre of a circle. TA and TB are tangents to the circle at A and B.

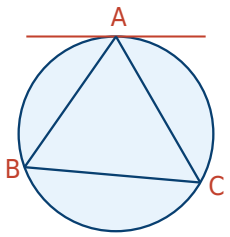


a. Angle $AOT = 62°$. Find angle ATO. [3 marks]
b. Angle $ATB = 50°$. Find angle TAB. [2 marks]
c. Using the same figure, find angle OAB and angle AOB. [3 marks]

Question 6

Question 6 – The alternate segment theorem *(Section A · Skills practice)*

PQ is a tangent to a circle at A. B and C are points on the circle. Angle $PAB = 62°$ and angle $QAC = 48°$.

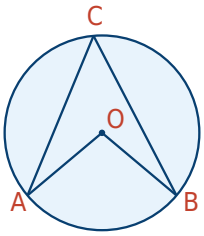


a. Find angle ACB and give a reason. [2 marks]
b. Find angle ABC. [2 marks]
c. Find angle BAC. [2 marks]

Question 7

Question 7 – The circular lake *(Section B · Real-life practice)*

Three posts A, B and C stand on the edge of a circular lake. A buoy O marks the centre. The angle at the buoy between the lines to A and B is $112°$.

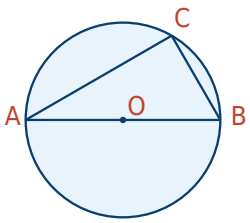


a. A walker stands at C. What angle does she see between the lines of sight to A and B? [2 marks]
b. A second walker stands at D on the shore, on the other side of AB from C. Find the angle ADB. [3 marks]
c. Explain why every walker standing on the arc ACB sees the same angle. [2 marks]

Question 8

Question 8 – Cutting a pizza *(Section B · Real-life practice)*

A circular pizza has diameter 30 cm. A straight cut AC goes from one point A on the edge to another point C on the edge, and the diameter is AB. The cut AC is 18 cm long.



a. Explain why angle ACB is a right angle. [1 mark]
b. Work out the length BC. [3 marks]
c. Work out the area of triangle ABC. [2 marks]
d. The pizza is cut into 8 equal slices. What is the angle at the centre of each slice? [1 mark]

Question 9

Question 9 – A lamp post beside a pond *(Section B · Real-life practice)*

A circular pond has centre O and radius 6 m. A lamp post at T is 10 m from O. A string is pulled tight from T to touch the pond at A so that TA is a tangent.

a. Explain why triangle OAT has a right angle at A. [1 mark]
b. Work out the length TA. [3 marks]
c. A second string touches the pond at B on the other side. What is the length TB and why? [2 marks]
d. Find angle ATO to 1 decimal place. [2 marks]

Question 10

Question 10 – A chord in a roundabout *(Section B · Real-life practice)*

A circular roundabout has radius 10 m. A straight path is cut across it as a chord AB of length 16 m. M is the midpoint of AB and O is the centre of the roundabout.

a. Explain why OM is perpendicular to AB. [2 marks]
b. Work out the distance OM. [3 marks]
c. Work out the area of triangle OAB. [2 marks]
d. Work out angle AOB to 1 decimal place. [3 marks]

Question 11

Question 11 – The stone arch *(Section B · Real-life practice)*

A bridge has a circular arc. The chord at water level is 24 m wide and the highest point of the arch is 4 m above the water. The centre O of the circle is below the water.

a. Let the radius be r. Write down the distance from O to the chord. [1 mark]
b. Use Pythagoras to show that $r^2 = 12^2 + (r-4)^2$. [2 marks]
c. Solve the equation to find r. [3 marks]

d. How far below the water is the centre O? [1 mark]

Question 12

Question 12 – A cyclic quadrilateral with algebra *(Section B · Real-life practice)*

A circular stage has four spotlights at A, B, C and D on its edge, in that order. Angle $DAB = (4x + 5)^\circ$ and angle $BCD = (2x + 25)^\circ$.

a. Explain why $4x + 5 + 2x + 25 = 180$. [1 mark]
b. Solve the equation. [2 marks]
c. Find both angles. [2 marks]
d. Angle $ABC = 100^\circ$. Find angle ADC and explain how you know. [2 marks]

Question 13

Question 13 – The lighthouse *(Section B · Real-life practice)*

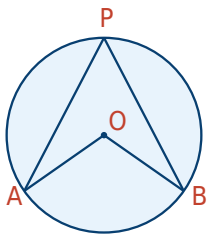
A lighthouse is at the centre O of a circular danger zone of radius 5 km. A ship at S is 13 km from the lighthouse and sails along a straight line that just touches the edge of the zone at P.

a. Explain why angle $OPS = 90^\circ$. [1 mark]
b. Work out how far the ship has sailed from S to P. [3 marks]
c. Find angle OSP to 1 decimal place. [2 marks]
d. The ship sails at 24 km/h. How long does it take to reach P in minutes? [2 marks]

Question 14

Question 14 – The best shooting position *(Section B · Real-life practice)*

A football goal has posts at A and B. A coach marks a circle through A and B on the pitch. A striker at P on the circle sees the goal under an angle $APB = 40^\circ$. The circle has centre O.



a. Work out angle AOB and give a reason. [2 marks]
b. A second striker stands at Q on the same arc as P. What angle does she see? [2 marks]
c. A third striker stands at R on the other side of AB on the circle. Work out angle ARB. [2 marks]
d. The goal is 7.32 m wide. Work out the radius of the circle to 2 decimal places. [3

marks]

Question 15

Question 15 – Isosceles triangles and a tangent *(Section C · Problem solving)*

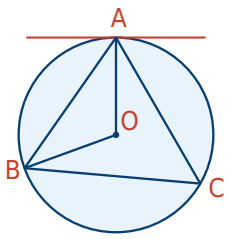
O is the centre of a circle. Triangle OAB has $OA = OB$ (radii) and angle $AOB = 100°$. TA and TB are tangents to the circle at A and B.

a. Find angle OAB and explain how you know. [2 marks]
b. Find angle TAB. [2 marks]
c. Find angle ATB. [2 marks]
d. Show that $TA = TB$ and describe quadrilateral $OATB$. [3 marks]

Question 16

Question 16 – Alternate segment and angle at the centre *(Section C · Problem solving)*

A, B and C lie on a circle with centre O. TA is a tangent at A. Angle $TAB = 64°$.



a. Find angle ACB and give a reason. [2 marks]
b. Find angle AOB and give a reason. [2 marks]
c. Find angle OAB and show that it agrees with the tangent-radius angle. [4 marks]

Question 17

Question 17 – Proving the cyclic quadrilateral theorem *(Section C · Problem solving)*

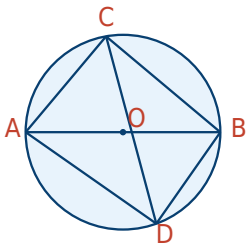
A, B, C and D are on a circle with centre O. Angle $ABC = x$.

a. Write the reflex angle AOC in terms of x and give a reason. [2 marks]
b. Write the angle AOC (the other angle at O) in terms of x. [2 marks]
c. Write angle ADC in terms of x and so prove that opposite angles of a cyclic quadrilateral add to $180°$. [4 marks]

Question 18

Question 18 – Two circle theorems in one problem *(Section C · Problem solving)*

AB is a diameter of a circle with centre O. C and D are points on the circle on opposite sides of AB. Angle $CAB = 28°$ and angle $ABD = 35°$.



a. Find angle ACB. [1 mark]
b. Find angle ABC. [1 mark]
c. Find angle BAD, giving a reason. [3 marks]
d. Find angle ADC, giving reasons. [4 marks]

Question 19

Question 19 – Chord, tangent and Pythagoras *(Section C · Problem solving)*

Two circles have the same centre O. The radius of the smaller circle is 5 cm and the radius of the larger circle is 13 cm. A chord AB of the larger circle touches the smaller circle at M.

a. Explain why M is the midpoint of AB. [2 marks]
b. Work out the length AB. [3 marks]
c. Work out the area of the ring between the two circles in terms of π. [2 marks]

Question 20

Question 20 – Challenge: the largest triangle *(Section D · Challenge)*

AB is the diameter of a circle and C is on the circle with $AC = 10$ cm and $BC = 24$ cm.

a. Show that the radius of the circle is 13 cm. [3 marks]
b. Find the shortest distance from C to the diameter AB. [4 marks]
c. Explain why no point on the circle gives a triangle with base AB and area greater than 13×13 cm². [3 marks]