Answers - Circle theorems

Year: Year 11

Answer to Question 1

a. $AOB = 2 \times 38 = 76^\circ$. Reason: the angle at the centre is twice the angle at the circumference.
b. $ACB = 130 \div 2 = 65^\circ$. Reason: the angle at the circumference is half the angle at the centre.
c. $3x - 20 = 2x$, so $x = 20$. Check: $AOB = 40^\circ$ and $ACB = 20^\circ$, and $40 = 2 \times 20$. The angle at the centre is twice the angle at the circumference, so $AOB = 2 \times ACB$.

Answer to Question 2

a. $ACB = 90^\circ$. Reason: the angle in a semicircle is a right angle.
b. $ABC = 180 - 90 - 34 = 56^\circ$ (angles in a triangle add to 180°).
c. $MA = MC$ (radii), so triangle AMC is isosceles with base angles 34°. Angle $AMC = 180 - 34 - 34 = 112^\circ$. (Check: this is $2 \times 56^\circ$, twice angle ABC.)

Answer to Question 3

a. $ACD = 42^\circ$. Reason: angles in the same segment are equal (both stand on chord AD). Name the chord each angle stands on: angle ABD and angle ACD both stand on AD, so they are equal.
b. $CBD = 35^\circ$ (both CBD and CAD stand on chord CD). Angles subtended by the same arc at the circumference are equal.
c. $BDC = 28^\circ$ (both BDC and BAC stand on chord BC). So the three missing angles are 42°, 35° and 28°.

Answer to Question 4

a. Opposite angles of a cyclic quadrilateral add to 180°: $C = 180 - 108 = 72^\circ$. Angles A and C are opposite each other in the cyclic quadrilateral.
b. $D = 180 - 85 = 95^\circ$. Angles B and D are also opposite, so they add up to 180°.
c. $2x + 10 + 3x - 20 = 180$, so $5x - 10 = 180$ and $5x = 190$, $x = 38$. $B = 2 \times 38 + 10 = 86^\circ$ and $D = 3 \times 38 - 20 = 94^\circ$. Check: $86 + 94 = 180$. Opposite angles B and D add up to 180°, so we add the two expressions and set the total equal to 180.

Answer to Question 5

a. The tangent meets the radius at 90°, so angle $OAT = 90^\circ$. Then $ATO = 180 - 90 - 62 = 28^\circ$.
b. Tangents from a point are equal, so $TA = TB$ and triangle TAB is isosceles: $TAB = (180 - 50) \div 2 = 65^\circ$.
c. $OAB = 90 - 65 = 25^\circ$. In the kite $OATB$: $AOB = 360 - 90 - 90 - 50 = 130^\circ$. The angles of a quadrilateral add up to 360°.

Answer to Question 6

a. $ACB = 62^\circ$. Reason: the angle between a tangent and a chord equals the angle in the alternate segment.
b. $ABC = 48^\circ$ (alternate segment theorem with angle QAC).
c. $BAC = 180 - 62 - 48 = 70^\circ$ (angles on a straight line at A, or angles in triangle ABC).

Answer to Question 7

a. $ACB = 112 \div 2 = 56^\circ$ (angle at the centre is twice the angle at the circumference).
b. D is on the opposite arc, so $ACBD$ is a cyclic quadrilateral. $ADB = 180 - 56 = 124^\circ$ (opposite angles add to 180°).
c. Angles in the same segment are equal: every point on the arc ACB is in the same segment and stands on the chord AB, so each sees 56°.

Answer to Question 8

a. AB is a diameter, so the angle in a semicircle is 90°. So the cut and the two halves of the diameter make a right-angled triangle with the right angle at C.
b. Pythagoras: $BC^2 = 30^2 - 18^2 = 900 - 324 = 576$, so $BC =$ **24 cm**. The diameter $AB = 30$ cm is the hypotenuse because it is opposite the right angle.
c. Area $= \frac{1}{2} \times 18 \times 24 =$ **216 cm²**. The two shorter sides are perpendicular, so they are the base and the height of the triangle.
d. $360 \div 8 = 45^\circ$. The 8 equal slices share the full turn of 360° at the centre.

Answer to Question 9

a. The radius OA meets the tangent TA at 90°. A tangent is always perpendicular to the radius at the point where it touches the circle.
b. $TA^2 = 10^2 - 6^2 = 100 - 36 = 64$, so $TA =$ **8 m**. $OT = 10$ m is the hypotenuse,

because it is opposite the right angle at A.

c. $TB = $ **8 m**: tangents drawn to a circle from the same point are equal in length.

d. $\sin(ATO) = \dfrac{6}{10} = 0.6$, so $ATO = 36.9^\circ$. In the right-angled triangle, OA is opposite angle ATO and OT is the hypotenuse, so we use sine.

Answer to Question 10

a. The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord (equivalently, $OA = OB$ makes triangle OAB isosceles and OM is its line of symmetry).

b. $AM = 8$ and $OA = 10$. $OM^2 = 100 - 64 = 36$, so $OM = $ **6 m**.

c. $\frac{1}{2} \times 16 \times 6 = $ **48 m²**.

d. $\sin(AOM) = \dfrac{8}{10}$, so $AOM = 53.13^\circ$ and $AOB = 2 \times 53.13 = 106.3^\circ$.

Answer to Question 11

a. O is $r - 4$ metres below the chord (the top of the arc is r above O, and the chord is 4 m below the top).

b. The half-chord is 12 m, the distance from O to the chord is $r - 4$ and the radius is r, so $r^2 = 12^2 + (r - 4)^2$.

c. $r^2 = 144 + r^2 - 8r + 16$, so $8r = 160$ and $r = 20$ **m**.

d. $20 - 4 = $ **16 m** below the water.

Answer to Question 12

a. $ABCD$ is a cyclic quadrilateral and opposite angles add to 180°.

b. $6x + 30 = 180$, so $6x = 150$ and $x = 25$. Collect the x terms and the numbers, then divide by 6.

c. $DAB = 4 \times 25 + 5 = 105^\circ$ and $BCD = 2 \times 25 + 25 = 75^\circ$. Check: $105 + 75 = 180$.

d. $ADC = 180 - 100 = 80^\circ$, since B and D are opposite angles of a cyclic quadrilateral.

Answer to Question 13

a. The ship's path is a tangent and the radius to the point of contact is perpendicular to it.

b. $SP^2 = 13^2 - 5^2 = 144$, so $SP = $ **12 km**. The distance $OS = 13$ km is the hypotenuse, so we subtract the squares.

c. $\sin(OSP) = \dfrac{5}{13}$, so $OSP = 22.6^\circ$. OP is opposite angle OSP and OS is the

hypotenuse, so we use sine.

d. $\frac{12}{24} = 0.5$ hours $=$ **30 minutes**. Time $=$ distance $\div$ speed, then multiply by 60 to change hours into minutes.

Answer to Question 14

a. $AOB = 2 \times 40 = 80^\circ$ (angle at the centre is twice the angle at the circumference).
b. 40°: angles in the same segment are equal.
c. $ARB = 180 - 40 = 140^\circ$ (opposite angles of a cyclic quadrilateral).
d. Triangle OAB is isosceles with apex angle 80°. The perpendicular from O bisects AB: half-width 3.66 m and half-angle 40°. $r = \frac{3.66}{\sin 40^\circ} =$ **5.69 m**.

Answer to Question 15

a. Triangle OAB is isosceles ($OA = OB$), so $OAB = (180 - 100) \div 2 = 40^\circ$.
b. The tangent is perpendicular to the radius, so $OAT = 90^\circ$ and $TAB = 90 - 40 = 50^\circ$.
c. Quadrilateral $OATB$: $ATB = 360 - 90 - 90 - 100 = 80^\circ$.
d. Tangents from a point are equal: $TA = TB$. $OATB$ has two pairs of equal adjacent sides ($OA = OB$ and $TA = TB$), so it is a **kite**.

Answer to Question 16

a. $ACB = 64^\circ$ (alternate segment theorem).
b. $AOB = 2 \times 64 = 128^\circ$ (angle at the centre is twice the angle at the circumference).
c. $OA = OB$ so triangle OAB is isosceles: $OAB = (180 - 128) \div 2 = 26^\circ$. The tangent-radius angle is $TAB + OAB = 64 + 26 = 90^\circ$, as required.

Answer to Question 17

a. The angle at the centre is twice the angle at the circumference, so the angle AOC standing on the arc ADC is $2x$.
b. Angles around a point add to 360°, so the other angle AOC (on the arc ABC) is $360 - 2x$.
c. ADC stands on the arc ABC, so $ADC = \frac{1}{2}(360 - 2x) = 180 - x$. Then $ABC + ADC = x + (180 - x) = 180^\circ$. Opposite angles of a cyclic quadrilateral add to 180°.

Answer to Question 18

a. $ACB = 90^\circ$ (angle in a semicircle).
b. $ABC = 180 - 90 - 28 = 62^\circ$.
c. $ADB = 90^\circ$ (angle in a semicircle), so $BAD = 180 - 90 - 35 = 55^\circ$.
d. $CDB = CAB = 28^\circ$ (angles in the same segment on chord CB). Then $ADC = ADB - CDB = 90 - 28 = 62^\circ$. This agrees with $ABC = 62^\circ$, because ADC and ABC are angles in the same segment on chord AC.

Answer to Question 19

a. AB is a tangent to the smaller circle at M, so $OM \perp AB$. The perpendicular from the centre to a chord bisects the chord, so $AM = MB$.
b. $AM^2 = 13^2 - 5^2 = 144$, so $AM = 12$ and $AB =$ **24 cm**. In the right-angled triangle OMA, the hypotenuse is the radius of the larger circle and OM is the radius of the smaller one.
c. Area $= \pi \times 13^2 - \pi \times 5^2 = 169\pi - 25\pi = 144\pi$ **cm²** (which is $\pi \times AM^2$).

Answer to Question 20

a. Angle $ACB = 90^\circ$ (semicircle), so $AB^2 = 10^2 + 24^2 = 676$, $AB = 26$ and the radius is **13 cm**.
b. Area of $ABC = \frac{1}{2} \times 10 \times 24 = 120$. Also area $= \frac{1}{2} \times 26 \times h$, so $13h = 120$ and $h = \dfrac{120}{13} =$ **9.23 cm**.
c. The height of any triangle on base AB is at most the radius (13 cm), reached when C is at the top of the circle directly above O. So the area is at most $\frac{1}{2} \times 26 \times 13 = 169$ cm², i.e. 13×13.