Answers - Charts, averages and cumulative frequency

Year: Year 11

Answer to Question 1

a. 3, 3, 3, 5, 7, 8, 9, 10, 12.
b. Total $= 60$, so the mean is $60 \div 9 =$ **6.67** minutes.
c. The middle (5th) value is **7**. The most common value is **3**.
d. $12 - 3 =$ **9** minutes.
e. The mode (3 minutes) because it is the most common value and is lower than the mean.

Answer to Question 2

a. $0 \times 6 + 1 \times 9 + 2 \times 7 + 3 \times 5 + 4 \times 3 = 0 + 9 + 14 + 15 + 12 =$ **50** pets.
b. $50 \div 30 =$ **1.67**.
c. With 30 values the median is halfway between the 15th and 16th. The cumulative frequencies are 6, 15, 22, 27, 30. The 15th value is 1 and the 16th is 2, so the median is **1.5**.
d. The highest frequency is 9, so the mode is **1**.

Answer to Question 3

a. The class with the highest frequency: $20 < t \leq 30$.
b. Midpoints: 5, 15, 25, 35.
$5 \times 4 + 15 \times 9 + 25 \times 12 + 35 \times 5 = 20 + 135 + 300 + 175 = 630$. Mean $\approx 630 \div 30 =$ **21 minutes**.
c. The median is between the 15th and 16th values. Cumulative frequencies: 4, 13, 25, 30. So the median is in $20 < t \leq 30$.
d. Grouped data does not give the exact values, so we assume each value is at the midpoint of its class.

Answer to Question 4

a. $360 \div 40 = 9^\circ$ per pupil. Football: $18 \times 9 = 162^\circ$. Netball: $12 \times 9 = 108^\circ$. Swimming: $6 \times 9 = 54^\circ$. Tennis: $4 \times 9 = 36^\circ$.
b. $162 + 108 + 54 + 36 = 360$.
c. $\frac{18}{40} = \frac{9}{20}$.

Answer to Question 5

a. Key: $2 \mid 1$ means 21 seconds.
b. The median is halfway between the 6th and 7th values: $(33 + 36) \div 2 =$ **34.5 seconds**.
c. Range $= 52 - 21 =$ **31 seconds**. Mode $=$ **25 seconds**.
d. Stems 2 and 3 each have four leaves.

Answer to Question 6

a. Frequency density $= \dfrac{\text{frequency}}{\text{class width}}$. Widths: 10, 20, 30, 40. Densities: $20 \div 10 =$ **2**, $50 \div 20 =$ **2.5**, $45 \div 30 =$ **1.5**, $40 \div 40 =$ **1**.
b. $20 + 50 + 45 + 40 =$ **155**.
c. More than 30 minutes: $45 + 40 = 85$. $85 \div 155 \times 100 =$ **54.8%**.

Answer to Question 7

a. 11A: range $88 - 20 =$ **68**; IQR $69 - 55 =$ **14**. 11B: range $92 - 12 =$ **80**; IQR $69 - 44 =$ **25**.
b. Class 11A has the higher median (62 against 58), so on average they did better. Class 11A also has the smaller IQR (14 against 25), so their marks are more consistent.
c. The highest mark is a single extreme value. It does not describe the whole class. The median and the spread are better measures, and they favour 11A.

Answer to Question 8

a. Cumulative frequencies: **6, 28, 62, 77, 80** (up to 150, 160, 170, 180, 190).
b. The median is the 40th value, in the class $160 < h \leq 170$: $160 + \dfrac{40 - 28}{34} \times 10 =$ **163.5 cm**.
c. LQ is the 20th value: $150 + \dfrac{20 - 6}{22} \times 10 =$ **156.4 cm**. UQ is the 60th value: $160 + \dfrac{60 - 28}{34} \times 10 =$ **169.4 cm**. IQR $= 169.4 - 156.4 =$ **13.0 cm**.
d. At 175 cm the cumulative frequency is about $62 + \frac{1}{2} \times 15 = 69.5$. So about $80 - 69.5 \approx$ **10 or 11 pupils** are taller than 175 cm.

Answer to Question 9

a. Each box plot has a box from the lower to the upper quartile with a line at the median, and whiskers to the fastest and slowest times. Ben: box from 12.5 to 13.4, median 12.9. Tara: box from 12.6 to 12.9, median 12.7.
b. Ben: range $14.2 - 12.1 =$ **2.1 s**, IQR $13.4 - 12.5 =$ **0.9 s**. Tara: range $13.1 - 12.4 =$ **0.7 s**, IQR $12.9 - 12.6 =$ **0.3 s**.
c. **Tara** is better overall: her median time (12.7) is lower than Ben's (12.9), and her times are far more consistent (smaller range and IQR). Ben has the single fastest time, but also the slowest.

Answer to Question 10

a. $5 \times 64 =$ **320**.
b. $(320 + 70) \div 6 = 390 \div 6 =$ **65**.
c. Total needed $= 7 \times 67 = 469$. After six tests the total is 390. So she needs $469 - 390 =$ **79**.
d. Sorted: 48, 59, 62, 70, 73, 78. Median $= (62 + 70) \div 2 = 66$. Mean $= 390 \div 6 = 65$. **True**: 66 is higher than 65.

Answer to Question 11

a. Total $= 18 + 20 + 20 + 22 + 22 + 24 + 25 + 28 + 31 + 150 = 360$. Mean $= 360 \div 10 =$ **£36 000**.
b. Median $= (22 + 24) \div 2 =$ **£23 000**. The modes are **£20 000 and £22 000** (both appear twice).
c. The owner's very large salary pulls the mean up. Nine of the ten workers earn less than £36 000, so the mean is not typical.
d. The **median** (£23 000) because it is not affected by extreme values.

Answer to Question 12

a. The highest frequency is 6, so the mode is **42 cups**.
b. There are 20 days, so the median is halfway between the 10th and 11th. Cumulative frequencies: 3, 8, 14, 18, 20. Both the 10th and 11th are 42, so the median is **42**.
c. $40 \times 3 + 41 \times 5 + 42 \times 6 + 43 \times 4 + 44 \times 2 = 120 + 205 + 252 + 172 + 88 = 837$. Mean $= 837 \div 20 =$ **41.85**.
d. $41.85 \times 1.80 =$ **£75.33** (to the nearest penny).

Answer to Question 13

a. Total $= 33$, mean $= 33 \div 7 =$ **4.71 mm** (2 d.p.). Sorted: 0, 0, 3, 4, 5, 9, 12, so the median is **4 mm**. Range $= 12 - 0 =$ **12 mm**.

b. The tallest bar is the third day (**12 mm**).
c. $7 \times 6.0 =$ **42 mm**.
d. Yes, because $6.0 > 4.71$, so the second week was wetter on average.

Answer to Question 14

a. $40 < s \leq 50$ (highest frequency, 30).
b. Midpoints 25, 35, 45, 55, 65: $200 + 840 + 1350 + 770 + 260 = 3420$. Mean $\approx 3420 \div 80 =$ **42.75 mph**.
c. Above 50 mph: $14 + 4 = 18$ cars. $18 \div 80 \times 100 =$ **22.5%**.
d. The data is grouped, so we do not know where inside the class $50 < s \leq 60$ each car lies.

Answer to Question 15

a. Frequency = frequency density × class width: $0.5 \times 20 =$ **10**; $2.4 \times 10 =$ **24**; $3.2 \times 10 =$ **32**; $1.4 \times 20 =$ **28**; $0.3 \times 40 =$ **12**.
b. $10 + 24 + 32 + 28 + 12 =$ **106** customers.
c. Between 25 and 30: half of 24 = 12. Between 30 and 40: 32. Between 40 and 50: half of 28 = 14. Total $= 12 + 32 + 14 =$ **58** customers.
d. The median is the 53rd value. Cumulative: 10, 34, 66. It is in $30 < a \leq 40$: $30 + \dfrac{53 - 34}{32} \times 10 =$ **35.9 years**.

Answer to Question 16

a. Plot (10, 8), (20, 30), (30, 68), (40, 90), (50, 100), starting at (0, 0), and join with a smooth curve or straight lines.
b. Median is the 50th value: $20 + \dfrac{50 - 30}{38} \times 10 =$ **25.3 min**. LQ (25th): $10 + \dfrac{25 - 8}{22} \times 10 = 17.7$. UQ (75th): $30 + \dfrac{75 - 68}{22} \times 10 = 33.2$. IQR $= 33.2 - 17.7 =$ **15.5 min**.
c. At 35 minutes the cumulative frequency is about $68 + \frac{1}{2} \times 22 = 79$. This is 79%, which is just below 80%. The target was **not quite met**.

Answer to Question 17

a. The total frequency is $5 + 7 + x + 6 + 4 = 22 + x$. The total score is $5 + 14 + 3x + 24 + 20 = 63 + 3x$. The mean is 2.9, so $\dfrac{63 + 3x}{22 + x} = 2.9$. Multiply both sides by $22 + x$: $63 + 3x = 2.9(22 + x) = 63.8 + 2.9x$. Subtract $2.9x$ and 63:

$0.1x = 0.8$, so $x = 8$. Check: total score $= 87$, $87 \div 30 = 2.9$.

b. Frequency 8 is the highest, so the mode is **3**.

c. 30 values: halfway between the 15th and 16th. Cumulative frequencies: 5, 12, 20, 26, 30. Both the 15th and 16th are 3, so the median is **3**.

d. $6 + 4 = 10$ pupils scored 4 or more. $10 \div 30 = \frac{1}{3}$ exactly, so it is **not over** a third.

Answer to Question 18

a. Class A: $24 \times 62 =$ **1 488**. Class B: $16 \times 72.5 =$ **1 160**.

b. Total $= 2648$ marks for 40 pupils. Mean $= 2648 \div 40 =$ **66.2**.

c. New total $= 2648 - 40 - 85 = 2523$. New number $= 38$. Mean $= 2523 \div 38 =$ **66.4** (1 d.p.).

Answer to Question 19

a. Year 9: $\frac{120}{360} \times 60 =$ **20 pupils**. Year 11: $\frac{100}{360} \times 90 =$ **25 pupils**.

b. **No.** A bigger angle means a bigger fraction of its own group, but Year 11 has more pupils overall, so more Year 11 pupils chose football.

c. The **total number of pupils** (or the frequency of each sector) for each chart.

Answer to Question 20

The total is $5 \times 7 = 35$. The median is 8, so the middle number is 8. The mode is 9, so 9 appears at least twice. Only the two largest numbers can be 9 and 9 (a smaller number cannot be 9 and still be in order, and any other repeat would be a different mode). So the list is $a,\ b,\ 8,\ 9,\ 9$. The range is 7, so $9 - a = 7$ and $a = 2$. The total gives $2 + b + 8 + 9 + 9 = 35$, so $b = 7$. The numbers are **2, 7, 8, 9, 9**. Check: mean $35 \div 5 = 7$, median 8, mode 9, range $9 - 2 = 7$. Each step was forced, so the answer is unique.