Answers - Units, estimation, rounding and bounds

Year: Year 10

Answer to Question 1

a. $3.5 \times 1000 =$ **3500 m** There are 1000 metres in a kilometre, so we multiply by 1000. So 3.5 km is 3500 m.
b. $2500 \div 1000 =$ **2.5 litres** There are 1000 ml in a litre, so we divide by 1000. So 2500 ml is 2.5 litres.
c. $0.8 \times 1000 =$ **800 kg** One tonne is 1000 kg, so we multiply by 1000. So 0.8 tonnes is 800 kg.
d. $1\ \text{m}^2 = 100 \times 100 = 10\,000\ \text{cm}^2$, so $4.5 \times 10\,000 =$ **45 000 cm²** For areas the conversion factor is squared, so we multiply by 10 000, not 100.

Answer to Question 2

a. **4.27** (2 d.p.) and **4.27** (3 s.f.) To 2 d.p., the third decimal digit 7 makes the 6 round up to 7. To 3 s.f. we keep 4, 2 and 6, and the next digit 7 rounds up.
b. The first two significant figures are 4 and 5, next digit 6, so **0.0046** The zeros at the start are not significant.
c. **25 500** (3 s.f.) and **25 500** (nearest 100) To 3 s.f. the fourth digit 8 makes the 4 round up to 5.
d. **8.0** (1 d.p.) and **8.0** (2 s.f.). The zero shows the accuracy. Rounding 7.996 makes the 9s carry over, giving 8.0.

Answer to Question 3

a. $5 \times 20 =$ **100** Round 4.8 to 5 and 19.7 to 20, then multiply. The estimate is 100.
b. $\dfrac{300}{0.5} =$ **600** Round 304 to 300 and 0.52 to 0.5. Dividing by 0.5 is the same as multiplying by 2, so the estimate is 600.
c. $\dfrac{60 \times 0.4}{0.2} =$ **120** Round 58.7 to 60, 0.41 to 0.4 and 0.19 to 0.2. Then $60 \times 0.4 = 24$ and $24 \div 0.2 = 120$. The estimate is 120.
d. $\sqrt{81} + \sqrt{25} = 9 + 5 =$ **14** 82 is close to the square number 81 and 26 is close to 25, so the square roots are about 9 and 5. The estimate is 14.

Answer to Question 4

a. $7.25 \leq x < 7.35$ The value was rounded to 1 decimal place, so it can be up to 0.05 below or above 7.3.
b. $445 \leq x < 455$ Rounding to 2 significant figures means rounding to the nearest 10, so the value can be up to 5 below or above 450.
c. $11.5 \leq x < 12.5$ Rounding to the nearest whole number means the value can be up to 0.5 below or above 12.
d. $3.4 \leq x < 3.5$ (truncating cuts off the digits, so the value is at least 3.4) So the error interval for a truncated value starts at the value itself and goes up to the next tenth.

Answer to Question 5

a. Half of 0.1 is 0.05: **8.35 and 8.45 cm** The lower bound is $8.4 - 0.05$ and the upper bound is $8.4 + 0.05$.
b. Half of 10 is 5: **245 and 255 g** The lower bound is $250 - 5$ and the upper bound is $250 + 5$.
c. **12.25 and 12.35 s** The time was measured to the nearest 0.1 s, so the bounds are 0.05 s either side of 12.3 s.
d. The accuracy is 0.1, half is 0.05: **2.45 and 2.55 litres**

Answer to Question 6

a. $15 \div 60 = 0.25$, so **2.25 hours** 15 minutes is a quarter of an hour, which is 0.25. So 2 hours 15 minutes is 2.25 hours.
b. $35 \div 5 = 7$ lots of 5 miles, $7 \times 8 =$ **56 km** 35 miles is 7 lots of 5 miles, so it is 7 lots of 8 km. So 35 miles is about 56 km.
c. $3.5 \times 2.2 =$ **7.7 lb** Each kilogram is about 2.2 pounds, so we multiply by 2.2. So 3.5 kg is about 7.7 lb.
d. $0.4 \times 60 = 24$, so **1 hour 24 minutes** Change the decimal part into minutes by multiplying by 60. So 1.4 hours is 1 hour 24 minutes.

Answer to Question 7

a. $50 \text{ mm} \div 500 =$ **0.1 mm** Change 5 cm into 50 mm, then divide by 500 sheets. One sheet is 0.1 mm thick.
b. **4.5 cm and 5.5 cm** (4.5 ≤ t < 5.5). The thickness was measured to the nearest centimetre, so it can be up to 0.5 cm below or above 5 cm.
c. Least: $45 \div 500 =$ **0.090 mm**. Greatest: $55 \div 500 =$ **0.110 mm**. Divide each bound in millimetres by the 500 sheets.
d. $120 \times 5 = 600$ cm = **6 m**. Each pack is about 5 cm thick, so the stack is about 6 m high.

Answer to Question 8

a. $100 \div 12.3 =$ **8.13 m/s**
b. Time between **12.25 s and 12.35 s**.
c. Greatest: $100 \div 12.25 = 8.163$ m/s. Least: $100 \div 12.35 = 8.097$ m/s.
d. To 2 s.f. the bounds are **8.2** and **8.1**, which differ, so **no**. To 1 significant figure both bounds round to **8 m/s**, so her speed is 8 m/s to 1 s.f.

Answer to Question 9

a. $4.2 \times 3.8 =$ **15.96 m²** Multiply the length by the width. The area is 15.96 m².
b. Lower: $4.15 \times 3.75 = 15.5625$. Upper: $4.25 \times 3.85 = 16.3625$. So **15.56 m² and 16.36 m²** (2 d.p.). For the lower bound multiply the two lower bounds; for the upper bound multiply the two upper bounds.
c. $16.3625 \times 18 =$ **£294.53** (nearest penny). The greatest cost uses the upper bound of the area.
d. The greatest possible area is 16.3625 m², which is less than 16.4, so **yes, it is certainly enough**.

Answer to Question 10

a. Round each number to 1 significant figure: 12 sections becomes 10, 480 people becomes 500, and 9 rows stays 9. Multiply: $10 \times 500 \times 9 = 45\,000$. The estimate of the crowd is **45 000 people**.
b. Put the decimal point after the first digit: $45\,000 = 4.5 \times 10\,000$. In standard form the estimate is 4.5×10^4.
c. Divide the estimate by the capacity: $\dfrac{4.5 \times 10^4}{5.2 \times 10^4} = \dfrac{4.5}{5.2} = 0.865$. Multiply by 100: about **87%** of the seats might be filled.
d. The accurate value is $12 \times 480 \times 9 = 51\,840$. The estimate is an **underestimate**, because 12 was rounded down to 10 and this rounding has a bigger effect than rounding 480 up to 500.

Answer to Question 11

a. $2 \times 454 =$ **908 g** Multiply the number of pounds by 454 g. The flour is 908 g.
b. $6 \times 28.4 =$ **170.4 g** Multiply the number of ounces by 28.4 g. The butter is 170.4 g.
c. $10 \times 28.4 =$ **284 ml** $=$ **0.284 litres** Multiply by 28.4 ml, then divide by 1000 to change into litres. The milk is 284 ml.
d. $1000 \div 454 =$ **2.2 lb** Divide the grams by 454 to change into pounds. So 1 kg is about 2.2 lb.

Answer to Question 12

a. **240 cm by 180 cm** There are 100 cm in a metre, so multiply each length by 100.
b. $240 \div 15 = 16$ across and $180 \div 15 = 12$ up, so $16 \times 12 = 192$ tiles. **Yes, they fit exactly.**
c. $192 \div 20 = 9.6$, so **10 boxes** (200 tiles). We round up because 9 boxes hold only 180 tiles, which is not enough.
d. Cost $= 10 \times 24 =$ **£240**. Left over $= 8$ tiles, which is $\frac{8}{200} =$ **4%**.

Answer to Question 13

a. $4.5 \leq d < 5.5$ **ml**
b. Least: $4.5 \times 21 = 94.5$ ml. Greatest: $5.5 \times 21 = 115.5$ ml. The total is 21 doses, because 3 doses a day for 7 days is $3 \times 7 = 21$.
c. No. The greatest total of 115.5 ml is more than 100 ml, so one bottle **might not be enough**.
d. Spoons are made to measure exact standard amounts, so there is less error.

Answer to Question 14

a. $28 \div 8 = 3.5$, so **4 minibuses** (round up: half a bus cannot be used).
b. $50 \div 12 = 4.17$, so **5 tins** (round up to have enough).
c. Money is paid to the nearest penny: **£47.38**.
d. 9.0×10^6 (the zero shows 2 significant figures).

Answer to Question 15

a. $6.45 \leq a < 6.55$ and $2.35 \leq b < 2.45$. Each value was rounded to 1 decimal place, so the bounds are 0.05 either side.
b. Upper bound of $a + b = 6.55 + 2.45 =$ **9.00**. Lower bound of $a - b = 6.45 - 2.45 =$ **4.00**. For the lower bound of a difference, subtract the upper bound of b from the lower bound of a.
c. Upper bound of $\frac{a}{b} = \frac{\text{UB } a}{\text{LB } b} = \frac{6.55}{2.35} =$ **2.787**. To make a fraction as large as possible, use the largest top and the smallest bottom.
d. Lower bound of $ab = 6.45 \times 2.35 =$ **15.1575**. Multiply the two lower bounds to get the smallest possible product.

Answer to Question 16

a. Distance **115 to 125 km**. Time **1.45 to 1.55 h**.
b. Greatest $= \frac{125}{1.45} =$ **86.2 km/h**.
c. Least $= \frac{115}{1.55} =$ **74.2 km/h**.
d. The bounds round to 90 and 70 to 1 s.f., which are different, so the speed **cannot** be given to 1 s.f. with certainty.

Answer to Question 17

a. Diameter **9.5 to 10.5 cm**, radius **4.75 to 5.25 cm**.
b. Upper: $\pi \times 5.25^2 = 27.5625\pi =$ **86.6 cm²**. Lower: $\pi \times 4.75^2 = 22.5625\pi =$ **70.9 cm²**.
c. The area could be anywhere from 70.9 to 86.6 cm², so the figure 78.5 (3 s.f.) suggests much more accuracy than is justified.
d. 70.9 and 86.6 differ in the first digit (70 and 90 to 1 s.f.), so **not even 1 significant figure** can be quoted with certainty.

Answer to Question 18

a. Rounded value **3.14** ($3.14159 \to 3.14$). Error interval $3.135 \leq x < 3.145$.
b. Truncated value **3.14**. Error interval $3.14 \leq x < 3.15$.
c. Both give 3.14, which differs from π by 0.0016, so they are **equally close** here.
d. Truncating 4.99 gives **4.9**, whereas rounding gives **5.0**. Truncating always cuts the number off and never rounds up.

Answer to Question 19

a. $|24.6 - 24.63| =$ **0.03 cm** The absolute error is the difference between the measured value and the true value. The error is 0.03 cm.
b. $\frac{0.03}{24.63} \times 100 =$ **0.12%** Divide the error by the true length and multiply by 100. The percentage error is 0.12%.
c. **0.05 cm** The rod was measured to the nearest 0.1 cm, so the measurement can be wrong by up to half of 0.1 cm, which is 0.05 cm.
d. $\frac{0.05}{24.6} \times 100 =$ **0.20%** Divide the greatest error by the measured length and multiply by 100. The greatest percentage error is 0.20%.

Answer to Question 20

a. **3.445 to 3.455 cm**.
b. Lower: $3.445^2 = 11.868025$. Upper: $3.455^2 = 11.937025$. So **11.8680 to 11.9370 cm²**. The bounds of the area are the squares of the bounds of the side.
c. Both round to 11.9 to 3 s.f.: **3 significant figures**. (To 4 s.f. they are 11.87 and 11.94.)
d. The area is **11.9 cm²** to 3 s.f.