Worksheet - Surds and exact values

Year: Year 10 · 20

Show your working.

Question 1

Question 1 – Answers in terms of pi *(Section A · Skills practice)*

Leave each answer in terms of π, then give a decimal to 1 decimal place.

a. the circumference of a circle with radius 5 cm
b. the area of a circle with diameter 12 cm
c. the area of a semicircle with radius 8 cm
d. the volume of a cylinder with radius 3 cm and height 10 cm

Question 2

Question 2 – What is a surd? *(Section A · Skills practice)*

A surd is a root that cannot be written as a fraction (it is irrational). Decide whether each number is a surd.

a. $\sqrt{16}$
b. $\sqrt{12}$
c. $\sqrt{\frac{9}{4}}$
d. $\sqrt{7}$ and π

Question 3

Question 3 – Simplifying surds *(Section A · Skills practice)*

Simplify each surd by finding the largest square factor. Higher tier.

a. $\sqrt{50}$
b. $\sqrt{72}$
c. $\sqrt{48}$
d. $\sqrt{200}$

Question 4

Question 4 – Adding and subtracting surds *(Section A · Skills practice)*

Simplify each surd first and then collect like terms. Higher tier.

a. $3\sqrt{2} + 5\sqrt{2}$
b. $\sqrt{12} + \sqrt{27}$
c. $\sqrt{50} - \sqrt{8}$
d. $2\sqrt{45} - \sqrt{20}$

Question 5

Question 5 – Multiplying and expanding surds *(Section A · Skills practice)*

Simplify or expand. Higher tier.

a. $\sqrt{3} \times \sqrt{12}$
b. $(2 + \sqrt{3})(2 - \sqrt{3})$
c. $(1 + \sqrt{5})^2$
d. $(3 - \sqrt{2})(3 + \sqrt{2})$

Question 6

Question 6 – Rationalising the denominator *(Section A · Skills practice)*

Rationalise the denominator of each fraction. Higher tier.

a. $\dfrac{6}{\sqrt{3}}$
b. $\dfrac{10}{\sqrt{5}}$
c. $\dfrac{1}{1 + \sqrt{2}}$
d. $\dfrac{3}{2 - \sqrt{3}}$

Question 7

Question 7 – The circular pond *(Section B · Real-life practice)*

A circular pond has a radius of 4 m. A path of width 1 m goes all the way round it. Give exact answers in terms of π.

a. Find the area of the pond.
b. Find the outer radius of the path and the area of the whole circle (pond and path).
c. Find the area of the path.
d. Find the circumference of the pond and the circumference of the outer edge of the path.

Question 8

Question 8 – The diagonal brace *(Section B · Real-life practice)*

A gate frame is a rectangle 3 m wide and 5 m tall. A diagonal brace goes from one corner to the opposite corner. Give exact answers.

a. Find the length of the brace as a surd in its simplest form.
b. A square gate has side 5 m. Find the length of its diagonal brace.
c. Write the answer to (b) to 1 decimal place.
d. Which brace is longer, and by how much? Give your answer to 2 decimal places.

Question 9

Question 9 – The triangular sail *(Section B · Real-life practice)*

A sail is a right-angled triangle with the two shorter sides 4 m and 6 m. Give exact answers.

a. Find the length of the longest side as a surd in its simplest form.
b. Find the area of the sail.
c. Find the perimeter as an exact value.
d. The sail is made of cloth that costs £12 per m². Work out the cost of the cloth.

Question 10

Question 10 – The equilateral triangle *(Section B · Real-life practice)*

A roof panel is an equilateral triangle with side 6 m. Give exact answers.

a. Use Pythagoras to find the height of the triangle. Write it in simplest surd form.
b. Find the exact area of the panel.
c. Find the area to 1 decimal place.
d. Four of these panels are used. Find the total area in exact form.

Question 11

Question 11 – Cylinder and cone *(Section B · Real-life practice)*

A cylindrical can has radius 3 cm and height 10 cm. A cone has radius 3 cm and vertical height 4 cm. Give exact answers in terms of π.

a. Find the volume of the can.
b. Find the volume of the cone.
c. Find the slant height of the cone.
d. Find the curved surface area of the cone.

Question 12

Question 12 – The cube diagonal *(Section B · Real-life practice)*

A cube has side 4 cm. Give exact answers.

a. Find the diagonal of a face, as a surd in simplest form.
b. Find the length of the space diagonal (corner to opposite corner through the cube).
c. Write your answer to (b) to 2 decimal places.
d. Show that the space diagonal is $\sqrt{3}$ times the side.

Question 13

Question 13 – The sphere *(Section B · Real-life practice)*

A beach ball has radius 6 cm. A hemisphere bowl has the same radius. Leave answers in terms of π.

a. Find the volume of the ball.
b. Find the surface area of the ball.
c. Find the volume of the bowl.
d. Find the volume of the ball as a decimal to the nearest cm³.

Question 14

Question 14 – The ramp *(Section B · Real-life practice)*

A wheelchair ramp rises 1 m over a horizontal distance of 12 m. Give answers to 2 decimal places.

a. Find the length of the ramp using Pythagoras, as a surd.
b. Write this length to 2 decimal places.
c. Find the gradient as a fraction and as a percentage.
d. The guidance says the gradient should not be steeper than 1 in 12. Does this ramp meet the guidance?

Question 15

Question 15 – Higher: showing exact results *(Section C · Problem solving)*

Higher tier. Show that each statement is true, with working that shows every step.

a. $(\sqrt{8}+\sqrt{2})^2 = 18$
b. $\sqrt{18}+\sqrt{8} = 5\sqrt{2}$
c. $\dfrac{4+\sqrt{12}}{2} = 2+\sqrt{3}$

d. $\dfrac{\sqrt{2}+1}{\sqrt{2}-1} = 3+2\sqrt{2}$

Question 16

Question 16 – Higher: area and perimeter with surds *(Section C · Problem solving)*

Higher tier. A rectangle has sides $(3+\sqrt{5})$ cm and $(3-\sqrt{5})$ cm.

a. Show that the area is 4 cm².
b. Find the perimeter.
c. Find the length of the diagonal, in simplest surd form.
d. Check the diagonal to 2 d.p. using the decimal side lengths.

Question 17

Question 17 – Higher: solving with surds *(Section C · Problem solving)*

Higher tier. Solve each equation, leaving answers in simplest surd form.

a. $x^2 = 20$
b. $(x+3)^2 = 7$
c. $2x^2 = 50$ (give both integer solutions)
d. $x^2 - 6x + 2 = 0$ using the formula

Question 18

Question 18 – Is the triangle right-angled? *(Section C · Problem solving)*

Higher tier. A triangle has sides $\sqrt{12}$ cm, $\sqrt{28}$ cm and $2\sqrt{10}$ cm.

a. Write each side in simplest surd form.
b. Show that the triangle is right-angled.
c. Find the area of the triangle in exact form.
d. Find the area to 2 d.p.

Question 19

Question 19 – Exact trigonometry values *(Section C · Problem solving)*

Higher tier. Use the exact values in this table.

Angle	sin	cos	tan
30°	$\dfrac{1}{2}$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{\sqrt{3}}$

Angle	sin	cos	tan
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

a. Find $\sin 30^\circ \times \cos 60^\circ$ exactly.
b. Show that $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ = 1$.
c. A ladder of length 6 m makes an angle of 60° with the ground. Find exactly how far up the wall it reaches.
d. Rationalise $\tan 30^\circ = \frac{1}{\sqrt{3}}$.

Question 20

Question 20 – Challenge: the square root of a surd expression *(Section D · Challenge)*

Higher tier. You want to find a and b (whole numbers) such that $(a + b\sqrt{2})^2 = 17 + 12\sqrt{2}$.

a. Expand $(a + b\sqrt{2})^2$.
b. Compare the rational parts and the surd parts to form two equations.
c. Solve to find a and b (try $ab = 6$).
d. Hence write down $\sqrt{17 + 12\sqrt{2}}$.