Answers - Surds and exact values

Year: Year 10

Answer to Question 1

a. $2\pi \times 5 = 10\pi$ **cm** $= 31.4$ cm Use $C = 2\pi r$ with $r = 5$. The exact answer is 10π cm, which is about 31.4 cm.

b. radius 6, area $= \pi \times 36 = 36\pi$ **cm²** $= 113.1$ cm² Halve the diameter to get the radius 6 cm, then use $A = \pi r^2$. The exact area is 36π cm².

c. $\frac{1}{2}\pi \times 64 = 32\pi$ **cm²** $= 100.5$ cm² A semicircle is half of a circle of area $\pi \times 8^2 = 64\pi$. The exact area is 32π cm².

d. $\pi \times 9 \times 10 = 90\pi$ **cm³** $= 282.7$ cm³ Use $V = \pi r^2 h$ with $r = 3$ and $h = 10$. The exact volume is 90π cm³. Exact answers in terms of π avoid rounding errors.

Answer to Question 2

a. $\sqrt{16} = 4$: **not a surd** (rational). 16 is a square number, so its square root is a whole number.

b. $\sqrt{12} = 3.4641\ldots$ does not end or repeat: **a surd**. 12 is not a square number, so $\sqrt{12}$ is irrational. We can simplify it to $2\sqrt{3}$, but it is still a surd.

c. $\sqrt{\frac{9}{4}} = \frac{3}{2}$: **not a surd**. Both 9 and 4 are square numbers, so the root is the fraction $\frac{3}{2}$.

d. $\sqrt{7}$ is a **surd**. π is irrational but is not a root, so it is **not a surd**. 7 is not a square number, so $\sqrt{7}$ is irrational and is a surd.

Answer to Question 3

a. $\sqrt{25 \times 2} = 5\sqrt{2}$ 25 is the largest square number that divides 50, and $\sqrt{25} = 5$ comes outside the root. So $\sqrt{50} = 5\sqrt{2}$.

b. $\sqrt{36 \times 2} = 6\sqrt{2}$ 36 is the largest square factor of 72; using 9×8 would not finish the job. So $\sqrt{72} = 6\sqrt{2}$.

c. $\sqrt{16 \times 3} = 4\sqrt{3}$ 16 is the largest square factor of 48, and $\sqrt{16} = 4$. So $\sqrt{48} = 4\sqrt{3}$.

d. $\sqrt{100 \times 2} = 10\sqrt{2}$ 100 is the largest square factor of 200. Check: $(10\sqrt{2})^2 = 100 \times 2 = 200$.

Answer to Question 4

a. $8\sqrt{2}$ These are like surds, so we add them like $3x + 5x = 8x$. The answer is $8\sqrt{2}$.

b. $2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$ Simplify each surd: $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$ and $\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$. Now they are like surds, so the answer is $5\sqrt{3}$.

c. $5\sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$ Simplify each surd: $\sqrt{50} = 5\sqrt{2}$ and $\sqrt{8} = 2\sqrt{2}$. The answer is $3\sqrt{2}$.

d. $6\sqrt{5} - 2\sqrt{5} = 4\sqrt{5}$ Simplify each surd: $\sqrt{45} = 3\sqrt{5}$, so $2\sqrt{45} = 6\sqrt{5}$, and $\sqrt{20} = 2\sqrt{5}$. The answer is $4\sqrt{5}$.

Answer to Question 5

a. $\sqrt{36} = \mathbf{6}$ Use $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$: $3 \times 12 = 36$, which is a square number. The answer is 6.

b. $4 - 3 = \mathbf{1}$ (difference of two squares) This has the form $(a + b)(a - b) = a^2 - b^2$, and $(\sqrt{3})^2 = 3$, so the surds cancel. The answer is 1.

c. $1 + 2\sqrt{5} + 5 = 6 + 2\sqrt{5}$ Write the square as $(1 + \sqrt{5})(1 + \sqrt{5})$. There are two middle terms of $\sqrt{5}$, and $\sqrt{5} \times \sqrt{5} = 5$. The answer is $6 + 2\sqrt{5}$.

d. $9 - 2 = \mathbf{7}$ This is again a difference of two squares: $3^2 - (\sqrt{2})^2 = 9 - 2$. The answer is 7.

Answer to Question 6

a. $\dfrac{6\sqrt{3}}{3} = 2\sqrt{3}$ Multiply the top and the bottom by $\sqrt{3}$, because $\sqrt{3} \times \sqrt{3} = 3$. Then simplify $\frac{6}{3} = 2$. The answer is $2\sqrt{3}$.

b. $\dfrac{10\sqrt{5}}{5} = 2\sqrt{5}$ Multiply the top and the bottom by $\sqrt{5}$, then simplify $\frac{10}{5} = 2$. The answer is $2\sqrt{5}$.

c. Multiply by $\dfrac{1 - \sqrt{2}}{1 - \sqrt{2}}$: $\dfrac{1 - \sqrt{2}}{1 - 2} = \sqrt{2} - 1$ Multiply by the conjugate $1 - \sqrt{2}$, so the bottom becomes $1^2 - (\sqrt{2})^2 = -1$. Dividing by -1 changes both signs. The answer is $\sqrt{2} - 1$.

d. Multiply by $\dfrac{2 + \sqrt{3}}{2 + \sqrt{3}}$: $\dfrac{3(2 + \sqrt{3})}{4 - 3} = 6 + 3\sqrt{3}$ Multiply by the conjugate $2 + \sqrt{3}$, so the bottom becomes $4 - 3 = 1$. The answer is $6 + 3\sqrt{3}$.

Answer to Question 7

a. $\pi \times 4^2 = 16\pi$ **m²** Use $A = \pi r^2$ with $r = 4$. The pond has area 16π m².
b. Outer radius $= 5$ m, area $= 25\pi$ m². The path adds 1 m to the radius, so the outer radius is $4 + 1 = 5$ m.
c. $25\pi - 16\pi = 9\pi$ **m²** (≈ 28.3 m²). The path is the ring between the two circles, so we subtract the area of the pond from the area of the whole circle.
d. Pond: $2\pi \times 4 = 8\pi$ **m**. Outer: $2\pi \times 5 = 10\pi$ **m**. Use $C = 2\pi r$ for each circle. The outer edge is 2π m longer than the edge of the pond.

Answer to Question 8

a. $\sqrt{3^2 + 5^2} = \sqrt{34}$ **m** (5.83 m) The brace is the hypotenuse of a right-angled triangle with sides 3 m and 5 m. 34 has no square factor, so $\sqrt{34}$ is already in its simplest form.
b. $\sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$ **m** The square's diagonal is the hypotenuse of a triangle with two sides of 5 m. Simplify $\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$.
c. $5\sqrt{2} =$ **7.1 m** Using $\sqrt{2} \approx 1.4142$, $5\sqrt{2} \approx 7.07$, which is 7.1 m to 1 decimal place.
d. $5\sqrt{2} - \sqrt{34} = 7.07 - 5.83 =$ **1.24 m** longer for the square gate. Change both lengths to decimals, then subtract.

Answer to Question 9

a. $\sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}$ **m** The longest side is the hypotenuse, so we use Pythagoras. Then simplify: $\sqrt{52} = \sqrt{4 \times 13} = 2\sqrt{13}$.
b. $\frac{1}{2} \times 4 \times 6 =$ **12 m²** The two shorter sides are perpendicular, so they are the base and the height. The area is 12 m².
c. $4 + 6 + 2\sqrt{13} = 10 + 2\sqrt{13}$ **m** (17.2 m) Add the three sides. The whole numbers and the surd cannot be combined, so the exact perimeter is $10 + 2\sqrt{13}$ m.
d. $12 \times 12 =$ **£144** Multiply the area by the price per square metre. The cloth costs £144.

Answer to Question 10

a. Half the base is 3 m: $h = \sqrt{36 - 9} = \sqrt{27} = 3\sqrt{3}$ **m** The height cuts the equilateral triangle into two right-angled triangles with hypotenuse 6 m and base 3 m. Then $\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$.
b. $\frac{1}{2} \times 6 \times 3\sqrt{3} = 9\sqrt{3}$ **m²** Use area $= \frac{1}{2} \times$ base $\times$ height with base 6 and height $3\sqrt{3}$. The exact area is $9\sqrt{3}$ m².

c. $9\sqrt{3} = \mathbf{15.6\ m^2}$ Using $\sqrt{3} \approx 1.732$, $9\sqrt{3} \approx 15.59$, which is 15.6 m².
d. $4 \times 9\sqrt{3} = 36\sqrt{3}\ \mathbf{m^2}$ Multiply the exact area by 4. The total area is $36\sqrt{3}$ m², about 62.4 m².

Answer to Question 11

a. $\pi \times 9 \times 10 = 90\pi\ \mathbf{cm^3}$ Use $V = \pi r^2 h$ with $r^2 = 9$ and $h = 10$. The volume of the can is 90π cm³.
b. $\frac{1}{3}\pi \times 9 \times 4 = 12\pi\ \mathbf{cm^3}$ Use $V = \frac{1}{3}\pi r^2 h$ with $r^2 = 9$ and $h = 4$. The volume of the cone is 12π cm³.
c. $\sqrt{3^2 + 4^2} = \mathbf{5\ cm}$ The radius, the vertical height and the slant height form a right-angled triangle. The slant height is 5 cm.
d. $\pi r l = \pi \times 3 \times 5 = 15\pi\ \mathbf{cm^2}$ Use the slant height $l = 5$, not the vertical height. The curved surface area is 15π cm².

Answer to Question 12

a. $\sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}\ \mathbf{cm}$ The face diagonal is the hypotenuse of a right-angled triangle with two sides of 4 cm. Then $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$.
b. $\sqrt{(4\sqrt{2})^2 + 4^2} = \sqrt{32 + 16} = \sqrt{48} = 4\sqrt{3}\ \mathbf{cm}$ The space diagonal is the hypotenuse of a right-angled triangle formed by a face diagonal and a vertical edge. Then $\sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$.
c. $4\sqrt{3} = \mathbf{6.93\ cm}$ Using $\sqrt{3} \approx 1.7321$, $4\sqrt{3} \approx 6.93$ cm.
d. $4\sqrt{3} = \sqrt{3} \times 4$ ✓ So the space diagonal is $\sqrt{3}$ times the side length of 4 cm. In general, a cube of side a has space diagonal $\sqrt{3a^2} = a\sqrt{3}$.

Answer to Question 13

a. $\frac{4}{3}\pi \times 216 = 288\pi\ \mathbf{cm^3}$ Use $V = \frac{4}{3}\pi r^3$ with $r^3 = 6^3 = 216$, and $\frac{4}{3} \times 216 = 288$. The volume of the ball is 288π cm³.
b. $4\pi \times 36 = 144\pi\ \mathbf{cm^2}$ Use $A = 4\pi r^2$ with $r^2 = 36$. The surface area is 144π cm².
c. $\frac{1}{2} \times 288\pi = 144\pi\ \mathbf{cm^3}$ A hemisphere is half a sphere, so its volume is half of 288π.
d. $288\pi = \mathbf{905\ cm^3}$ Using $\pi \approx 3.14159$, $288\pi \approx 904.8$. The ball has a volume of about 905 cm³.

Answer to Question 14

a. $\sqrt{12^2 + 1^2} = \sqrt{145}$ **m** The ramp is the hypotenuse of a right-angled triangle with sides 12 m and 1 m. 145 has no square factor, so $\sqrt{145}$ is the exact length.

b. **12.04 m** $\sqrt{145} = 12.0416\ldots$, so the ramp is 12.04 m long.

c. $\frac{1}{12}$ = **8.3%** The gradient is the rise divided by the horizontal distance: $1 \div 12 = 0.0833$, which is about 8.3%.

d. The gradient is exactly 1 in 12, so it **just meets** the guidance. A gradient of 1 in 12 is the steepest allowed, so this ramp is acceptable.

Answer to Question 15

a. $\sqrt{8} = 2\sqrt{2}$, so $(2\sqrt{2} + \sqrt{2})^2 = (3\sqrt{2})^2 = 9 \times 2 = 18$ ✓. Simplify $\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$ first, then add the like surds before squaring.

b. $3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$ ✓. Simplify each surd: $\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$ and $\sqrt{8} = 2\sqrt{2}$. Then add the like surds.

c. $\sqrt{12} = 2\sqrt{3}$, so $\frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}$ ✓. Simplify $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$, then divide each term on the top by 2.

d. Multiply top and bottom by $\sqrt{2} + 1$: $\frac{(\sqrt{2} + 1)^2}{2 - 1} = 2 + 2\sqrt{2} + 1 = 3 + 2\sqrt{2}$ ✓. The bottom becomes $(\sqrt{2})^2 - 1^2 = 1$. Expanding $(\sqrt{2} + 1)^2$ gives two middle terms of $\sqrt{2}$.

Answer to Question 16

a. $(3 + \sqrt{5})(3 - \sqrt{5}) = 9 - 5 =$ **4** ✓ The product has the form $(a + b)(a - b) = a^2 - b^2$, so the surds cancel and the area is a whole number.

b. $2\left[(3 + \sqrt{5}) + (3 - \sqrt{5})\right] = 2 \times 6 =$ **12 cm** The perimeter is twice the sum of the sides, and the $\sqrt{5}$ terms cancel.

c. $d^2 = (3 + \sqrt{5})^2 + (3 - \sqrt{5})^2 = 14 + 6\sqrt{5} + 14 - 6\sqrt{5} = 28$, so $d = \sqrt{28} = 2\sqrt{7}$ **cm** Use Pythagoras. Each square has two middle terms $\pm 6\sqrt{5}$, which cancel. Then $\sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7}$.

d. Sides 5.236 and 0.764: $\sqrt{27.416 + 0.584} = \sqrt{28} = 5.29$ cm, and $2\sqrt{7} = 5.29$ ✓. Both methods give 5.29 cm, so the exact answer is correct.

Answer to Question 17

a. $x = \pm\sqrt{20} = \pm 2\sqrt{5}$ Take the square root of both sides, remembering $\pm$. Simplify $\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$.

b. $x + 3 = \pm\sqrt{7}$, so $x = -3 \pm \sqrt{7}$ Take the square root of both sides, remembering $\pm$, then subtract 3 from both sides.

c. $x^2 = 25$, so $x = \pm 5$ Divide both sides by 2 first, then take the square root. 25 is a square number, so the solutions are whole numbers.

d. $x = \dfrac{6 \pm \sqrt{36 - 8}}{2} = \dfrac{6 \pm \sqrt{28}}{2} = \dfrac{6 \pm 2\sqrt{7}}{2} = 3 \pm \sqrt{7}$ Use the formula with $a = 1$, $b = -6$, $c = 2$. Simplify $\sqrt{28} = 2\sqrt{7}$, then divide each term by 2.

Answer to Question 18

a. $2\sqrt{3}$, $2\sqrt{7}$, $2\sqrt{10}$ Use the largest square factors: $\sqrt{12} = \sqrt{4 \times 3}$, $\sqrt{28} = \sqrt{4 \times 7}$, and $2\sqrt{10}$ is already simplified.

b. $12 + 28 = 40 = (2\sqrt{10})^2$, so by the converse of Pythagoras the triangle is **right-angled**. Square each side: $(2\sqrt{3})^2 = 12$, $(2\sqrt{7})^2 = 28$ and $(2\sqrt{10})^2 = 40$.

c. $\frac{1}{2} \times 2\sqrt{3} \times 2\sqrt{7} = 2\sqrt{21}$ **cm²** The two shorter sides are perpendicular, so they are the base and the height. Multiply $2 \times 2 = 4$ and $\sqrt{3} \times \sqrt{7} = \sqrt{21}$, then halve.

d. $2\sqrt{21} = 9.17$ cm². Using $\sqrt{21} \approx 4.5826$, the area is about 9.17 cm².

Answer to Question 19

a. $\dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4}$ Read both values from the table: $\sin 30^\circ = \frac{1}{2}$ and $\cos 60^\circ = \frac{1}{2}$. The exact answer is $\frac{1}{4}$.

b. $\dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2} + \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{3}{4} + \dfrac{1}{4} = 1$ ✓ Substitute the exact values from the table: $\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{3}{4}$ because $\sqrt{3} \times \sqrt{3} = 3$.

c. $6 \sin 60^\circ = 6 \times \dfrac{\sqrt{3}}{2} = 3\sqrt{3}$ **m** (5.2 m) The ladder is the hypotenuse and the height up the wall is opposite the 60° angle, so we use sine. The ladder reaches $3\sqrt{3}$ m up the wall.

d. $\dfrac{1}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$ Multiply the top and bottom by $\sqrt{3}$, so the denominator becomes 3. So $\tan 30^\circ = \frac{\sqrt{3}}{3}$.

Answer to Question 20

a. $a^2 + 2ab\sqrt{2} + 2b^2$. Write the square as $(a + b\sqrt{2})(a + b\sqrt{2})$; the last term is $b^2 \times 2 = 2b^2$.

b. $a^2 + 2b^2 = 17$ and $2ab = 12$, so $ab = 6$. Two expressions of the form $p + q\sqrt{2}$ are equal only if their whole-number parts and their $\sqrt{2}$ parts are equal.

c. Try $a = 3$, $b = 2$: $9 + 8 = 17$ ✓ and $3 \times 2 = 6$ ✓. The pairs with $ab = 6$ are 1×6, 2×3, 3×2 and 6×1; only $a = 3$, $b = 2$ also gives $a^2 + 2b^2 = 17$.

d. $\sqrt{17 + 12\sqrt{2}} = 3 + 2\sqrt{2}$. Check: $3 + 2.828 = 5.828$ and $\sqrt{17 + 16.97} = \sqrt{33.97} = 5.828$ ✓.