Lesson - Straight-line graphs

Year: Year 10

Straight-line graphs link algebra and geometry. At GCSE you plot lines, find gradients and intercepts, write the equation of a line from information, work with midpoints and distances, and recognise parallel lines. Perpendicular lines are Higher tier only. Questions appear on all three papers and often involve real-life contexts such as taxi fares.

Starter activity

Imagine walking up a hill and the road rises 1 m for every 10 m you travel along the ground. Draw this as a right-angled triangle. Now try a steeper hill where the road rises 2 m for every 10 m. How could you compare the steepness of the two with a single number? Convert both into fractions and percentages. Which is steeper? Link your answer to the word “gradient” and sketch both roads on a pair of axes.

Coordinates and plotting

Definition: coordinates and quadrants

Coordinates (x, y) give a position: across first, then up or down. The axes split the plane into four **quadrants**.

Method: plotting a straight line

Coordinates are written (x, y): go across first, then up or down. The axes divide the plane into four quadrants. To draw $y = 2x - 3$, make a table of values: for $x = -1, 0, 2$ the values of y are $-5, -3, 1$. Plot the points and join them with a ruler. Three points are enough: a mistake shows up if they are not in line.

- Horizontal lines have equations $y = k$; vertical lines have equations $x = k$.
- The lines $y = x$ and $y = -x$ are the diagonals through the origin.

The equation $y = mx + c$

Definition: gradient and y-intercept

The **gradient** is how steep a line is. The **y-intercept** is where the line crosses the y-axis.

Rule: the equation y = mx + c

m is the **gradient** (steepness) and c is the **y-intercept** (where the line crosses the y-axis). For $y = -2x + 5$: gradient -2, intercept $(0, 5)$. A negative gradient slopes

downwards from left to right. If the equation is not in this form, rearrange it: $2y + 4x = 10$ gives $2y = -4x + 10$, so $y = -2x + 5$.

Finding the gradient

Rule: the gradient of a line

Gradient $= \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{y_2 - y_1}{x_2 - x_1}$. For points $(1, 2)$ and $(5, 10)$: $\dfrac{10 - 2}{5 - 1} = \dfrac{8}{4} = 2$. On a graph, draw a large triangle and read the vertical and horizontal changes.

Finding the equation of a line

Method: finding the equation of a line

1. Find the gradient m from two points.
2. Substitute m and one point into $y = mx + c$ to find c, or use $y - y_1 = m(x - x_1)$.

Worked example 1

Line through $(2, 5)$ and $(6, 13)$: $m = \dfrac{8}{4} = 2$. Use $5 = 2 \times 2 + c$, so $c = 1$. The equation is $y = 2x + 1$.

Worked example 2

A line through $(0, 6)$ and $(3, 0)$: $m = \dfrac{0 - 6}{3 - 0} = -2$ and the intercept is 6, so $y = -2x + 6$.

Checking whether a point is on a line

Method: checking whether a point is on a line

Substitute the coordinates into the equation. $(3, 10)$ is on $y = 3x + 1$ because $3 \times 3 + 1 = 10$. Points $A(1, 3)$, $B(3, 7)$ and $C(6, 13)$ lie on one line if the gradients AB and BC are equal: $\frac{4}{2} = 2$ and $\frac{6}{3} = 2$, so they do.

Parallel lines

Rule: parallel lines

Parallel lines have the **same gradient**. A line parallel to $y = 4x - 1$ has equation

$y = 4x + c$. Through $(2, 3)$: $3 = 8 + c$, $c = -5$, so $y = 4x - 5$.

Midpoint and length of a segment

Rule: midpoint and length of a segment

Midpoint of (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. For $(-2, 3)$ and $(6, 9)$ the midpoint is $(2, 6)$. Length by Pythagoras: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. For $(-4, 2)$ and $(8, 7)$: $\sqrt{12^2 + 5^2} = \sqrt{169} = 13$.

Real-life graphs

Key fact: gradient and intercept in context

A taxi charges £3 plus £2 per km. The cost is $C = 2d + 3$. The gradient (2) is the cost per km and the intercept (3) is the fixed charge. A 7 km journey costs $2 \times 7 + 3 = £17$.

Perpendicular lines (Higher tier)

Rule: perpendicular lines (Higher tier)

Two lines are perpendicular when the product of their gradients is -1. If one gradient is m, the perpendicular gradient is $-\frac{1}{m}$: turn the fraction upside down and change the sign. The perpendicular to $y = 2x + 1$ has gradient $-\frac{1}{2}$. Through $(4, 1)$: $1 = -\frac{1}{2} \times 4 + c$, so $c = 3$ and $y = -\frac{1}{2}x + 3$. Lines $3y = x + 6$ (gradient $\frac{1}{3}$) and $y = -3x + 2$ are perpendicular because $\frac{1}{3} \times (-3) = -1$.

More worked examples in exam style

Worked example 3

A line passes through $(-1, 4)$ and $(3, -4)$. Find its equation [3 marks].

Gradient $= \frac{-4 - 4}{3 - (-1)} = \frac{-8}{4} = -2$. Use $4 = -2(-1) + c$, so $c = 2$. The equation is $y = -2x + 2$. Check with the second point: $-2(3) + 2 = -4$. Correct.

Worked example 4

The line L has equation $2y = 6x - 8$. Write down its gradient and the

coordinates of the point where it crosses the y-axis [2 marks].

Divide by 2: $y = 3x - 4$. Gradient 3 and the point $(0, -4)$.

Worked example 5

Is the line through $(0, 5)$ and $(4, 13)$ parallel to $y = 2x - 7$? [2 marks].

Gradient $= \frac{13-5}{4} = 2$. Same gradient, so yes.

Worked example 6 (reading a distance-time graph)

A straight line on a distance-time graph from $(0, 0)$ to $(2, 30)$ has gradient 15, so the speed is 15 km/h.

A horizontal line means the object is stationary.

Common mistakes

Watch out

- Subtracting the coordinates in different orders in the gradient formula.
- Reading x and y the wrong way round when plotting.
- Forgetting to rearrange to $y = \ldots$ before reading off the gradient: in $2y = 6x + 4$ the gradient is 3, not 6.
- Giving the perpendicular gradient with the wrong sign.

Exam technique

Exam technique

- Use a sharp pencil and a ruler, and extend lines across the grid.
- For a gradient from a graph, draw a large triangle and write the vertical and horizontal lengths on it.
- Always check the final equation by substituting one of the given points.

Straight lines and what they tell us

A straight-line graph describes a relationship in which one quantity changes at a constant rate. Its equation $y = mx + c$ contains two pieces of information: m is the gradient, how much y changes when x goes up by one, and c is the y-intercept, where the line crosses the vertical axis. In real situations the gradient is the rate of change, such as pounds per mile or kilometres per hour, and the intercept is the starting value, such as a fixed charge. To find the equation of a line you need either the gradient and one point, or two points. In the second case, first work out the gradient from the coordinates, then substitute one

point to find c. To check an answer, substitute the other point and see whether the equation is satisfied.

Worked example 7: equation from a graph

A line passes through $(0, -3)$ and $(2, 1)$. Find its equation and where it crosses the x-axis [3 marks].

The y-intercept is read from the first point: $c = -3$. The gradient is $\dfrac{1 - (-3)}{2 - 0} = 2$. So $y = 2x - 3$. At the x-axis $y = 0$, so $2x = 3$ and $x = 1.5$. The line crosses at $(1.5, 0)$. Check the second point: $2 \times 2 - 3 = 1$ ✓.

Worked example 8: rearranging into y = mx + c

Find the gradient and y-intercept of $4x + 2y = 10$ [2 marks].

Subtract $4x$: $2y = -4x + 10$. Divide by 2: $y = -2x + 5$. The gradient is -2 and the y-intercept is 5. A common error is to read the gradient as 4, which is the coefficient of x before rearranging.

Worked example 9: reading a real-life graph

The graph shows a delivery driver's distance from the depot. She is 10 km away at 08:00 and 40 km away at 09:30. Find her speed and the time she arrives at a town 100 km away [4 marks].

Speed is the gradient: $\dfrac{40 - 10}{1.5} = 20$ km/h. Distance after t hours from 08:00 is $10 + 20t$. When the distance is 100: $20t = 90$, so $t = 4.5$ hours, and she arrives at 12:30, assuming she does not stop.

Worked example 10: proving a quadrilateral

Show that A$(0, 1)$, B$(4, 3)$, C$(6, 7)$ and D$(2, 5)$ form a parallelogram [3 marks].

Gradient AB $= \dfrac{3 - 1}{4} = \dfrac{1}{2}$, gradient DC $= \dfrac{7 - 5}{4} = \dfrac{1}{2}$, so AB is parallel to DC.

Gradient BC $= \dfrac{7 - 3}{2} = 2$, gradient AD $= \dfrac{5 - 1}{2} = 2$, so BC is parallel to AD. Both pairs of opposite sides are parallel, so ABCD is a parallelogram.

Note: the midpoint and length formulae

The midpoint of (x_1, y_1) and (x_2, y_2) is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$: the mean of the coordinates. The length of the segment is found by Pythagoras: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Always sketch a small right-angled triangle to see the horizontal and vertical differences.

Watch out: sign errors in the gradient

Subtract the coordinates in the same order on top and on the bottom. $\frac{y_2 - y_1}{x_2 - x_1}$ is correct and $\frac{y_2 - y_1}{x_1 - x_2}$ gives the wrong sign. A line that goes down from left to right must have a negative gradient. Horizontal lines have gradient 0 and the form $y = k$, and vertical lines have the form $x = k$.

Did you know?

The idea of describing a position by two numbers, called coordinates, was developed by the French philosopher and mathematician René Descartes in the 1630s. A famous story says that he thought of it while lying in bed watching a fly crawl across the ceiling and realised that its position could be given by its distances from two walls. That is why the grid we use is called the Cartesian plane, from his Latin name Cartesius. Descartes' idea joined algebra and geometry for the first time: a line became an equation and an equation became a picture. This link is the foundation of computer graphics, mapping systems, and the satnavs that locate your car on a road.

Otto's tip

When you have two points, sketch them first. If the line goes down from left to right, your gradient must come out negative. If it does not, check your subtraction.