Answers - Straight-line graphs

Year: Year 10

Answer to Question 1

a. $2(-1) - 3 = -5$, -3, -1, 1, 3. So **−5, −3, −1, 1, 3**. Substitute each x value into $2x - 3$. The y values go up by 2 each time, which is the gradient.
b. Gradient **2**, y-intercept **−3** (the point $(0, -3)$). In $y = mx + c$ the number in front of x is the gradient and the constant is the y-intercept.
c. $y = 0$: $2x = 3$, so $(1.5, 0)$. Every point on the x-axis has $y = 0$, so we solve $2x - 3 = 0$.
d. $2 \times 5 - 3 = 7 \neq 8$, so **no**. Substitute $x = 5$: the line gives $y = 7$, not 8, so $(5, 8)$ is not on the line.

Answer to Question 2

a. $\frac{12 - 3}{4 - 1} = \frac{9}{3} =$ **3** The y value goes up by 9 while the x value goes up by 3, so the line rises 3 for each 1 across.
b. $\frac{-3 - 5}{2 - (-2)} = \frac{-8}{4} =$ **−2** Be careful with the signs: $2 - (-2) = 4$. A negative gradient means the line slopes down from left to right.
c. $\frac{4 - 4}{5 - 0} =$ **0** (a horizontal line) Both points have the same y value, so the line is horizontal and its gradient is 0.
d. The change in x is 0, so the gradient would involve dividing by zero: it is **undefined**. The equation is $x = 3$.

Answer to Question 3

a. $y = 3x - 2$ The gradient is $m = 3$, and the line passes through $(0, -2)$, so the y-intercept is $c = -2$.
b. $y = -\frac{1}{2}x + c$: $1 = -2 + c$, $c = 3$. $y = -\frac{1}{2}x + 3$ Substitute the point $(4, 1)$ into $y = -\frac{1}{2}x + c$ to find c. Check: $-\frac{1}{2} \times 4 + 3 = 1$.
c. $m = \frac{11 - 5}{3 - 1} = 3$, then $5 = 3 + c$, $c = 2$. $y = 3x + 2$ Find the gradient first, then substitute one of the points to find c. Check with the other point: $3 \times 3 + 2 = 11$.
d. $m = \frac{0 - 6}{3} = -2$ and $c = 6$. $y = -2x + 6$ The point $(0, 6)$ is on the y-axis, so $c = 6$. Check: $-2 \times 3 + 6 = 0$.

Answer to Question 4

a. $y = 3x + 5$ **and** $2y = 6x + 1$ (which is $y = 3x + 0.5$) have gradient 3. Parallel lines have the same gradient. Rearrange $2y = 6x + 1$ by dividing by 2 before comparing.
b. $y = 3x + c$: $7 = 6 + c$, $c = 1$. $y = 3x + 1$ Use the same gradient 3 and substitute the point $(2, 7)$ to find c. Check: $3 \times 2 + 1 = 7$.
c. $y = -2x + c$: $1 = -2 + c$, $c = 3$. $y = -2x + 3$ Use the gradient -2 and substitute $(1, 1)$. Check: $-2 + 3 = 1$.
d. $y = -3x + 9$, gradient **−3**. Subtract $3x$ from both sides to make y the subject. The gradient is the coefficient of x.

Answer to Question 5

a. Midpoint $(5, 7)$. Length $\sqrt{6^2 + 8^2} = \sqrt{100} =$ **10** The midpoint is the mean of the x values and the mean of the y values. For the length, the changes are 6 across and 8 up, so we use Pythagoras.
b. Midpoint $(1.5, 8)$. Length $\sqrt{5^2 + 12^2} = \sqrt{169} =$ **13** Midpoint: $\left(\frac{-1+4}{2}, \frac{2+14}{2}\right)$. The changes are 5 across and 12 up.
c. Midpoint $(2.5, 6)$. Length $= \sqrt{25 + 144} =$ **13** Halve each coordinate of B for the midpoint. The changes are 5 and 12, so the length is 13.
d. Midpoint $(1, 2)$. Length $\sqrt{8^2 + 6^2} = \sqrt{100} =$ **10** Midpoint: $\left(\frac{-3+5}{2}, \frac{-1+5}{2}\right)$. The changes are 8 across and 6 up, so the length is 10.

Answer to Question 6

a. $4 \times 3 - 5 = 7$, **yes**. Substitute $x = 3$ into the equation: it gives $y = 7$, so the point lies on L.
b. $4 \times 2 - 5 = 3 \neq 4$, **no**. Substitute $x = 2$: the line gives $y = 3$, not 4, so the point is not on L.
c. $x = -1$: $y = -9$. $y = 11$: $4x = 16$, $x = 4$. Substitute $x = -1$ to find y. For $y = 11$, solve $4x - 5 = 11$.
d. y-axis: $(0, -5)$. x-axis: $4x = 5$, $x = 1.25$, so $(1.25, 0)$. The y-intercept is the constant term. On the x-axis $y = 0$, so solve $4x - 5 = 0$.

Answer to Question 7

a. The fixed charge of **£2.50** before any miles are driven. When $d = 0$, the cost is £2.50, so the intercept is the cost before the taxi moves.
b. The cost per mile, **£1.20 per mile**. Each extra mile adds £1.20 to the cost, so the gradient is the price per mile.

c. $2.5 + 1.2 \times 7.5 = 2.5 + 9 =$ **£11.50** Substitute $d = 7.5$ into the formula. The journey costs £11.50.
d. $20.5 - 2.5 = 18$, $18 \div 1.2 =$ **15 miles**. Subtract the fixed charge, then divide by the cost per mile. The journey was 15 miles.

Answer to Question 8

a. 8 km in 0.5 h: **16 km/h**.
b. The distance does not change, so she has **stopped for 15 minutes**.
c. 9 km in 45 minutes: $9 \div 0.75 =$ **12 km/h**.
d. 17 km in 1.5 h: $17 \div 1.5 =$ **11.3 km/h** (1 d.p.).

Answer to Question 9

a. $\frac{70 - 30}{6 - 2} =$ **10 litres per minute**. The gradient is the change in volume divided by the change in time: 40 litres in 4 minutes.
b. $V = 10t + c$: $30 = 20 + c$, $c = 10$. $V = 10t + 10$. Substitute the point $(2, 30)$ to find c. Check: $10 \times 6 + 10 = 70$.
c. At $t = 0$: **10 litres**. The start is $t = 0$, so the volume is the intercept c. The tank held 10 litres at the start.
d. $150 = 10t + 10$, $t =$ **14 minutes**. Subtract 10 from both sides, then divide by 10. The tank holds 150 litres after 14 minutes.

Answer to Question 10

a. $m = \frac{58}{50} =$ **1.16**. The line passes through the origin and the point $(50, 58)$, so the gradient is $58 \div 50$. Each pound is worth €1.16.
b. $E = 1.16P$ A straight line through the origin has $c = 0$, so the formula is $E = 1.16P$.
c. $325 \times 1.16 =$ **€377** Multiply the pounds by 1.16. £325 is worth €377.
d. $200 \div 1.16 =$ **£172.41** Rearrange to $P = E \div 1.16$. €200 is worth about £172.41.

Answer to Question 11

a. $\frac{12}{100} = \frac{3}{25}$ Divide 12 and 100 by their HCF, 4. So 12% is $\frac{3}{25}$.
b. $0.12 \times 450 =$ **54 m** The road rises 12 m for every 100 m, which is 0.12 m for every metre. The road rises 54 m.
c. $\frac{36}{400} = 0.09 =$ **9%** Divide the rise by the horizontal distance and change it into a

percentage. The gradient is 9%.
d. No, 9% is less than 12%, so it is **less steep**. A smaller percentage gradient means a gentler slope.

Answer to Question 12

a. Gradient **1.8** and intercept **32**. The equation is in the form $y = mx + c$, with $m = 1.8$ and $c = 32$.
b. $1.8 \times 20 + 32 =$ **68 °F** Substitute $C = 20$. So 20 °C is 68 °F.
c. $95 - 32 = 63$, $63 \div 1.8 =$ **35 °C** Subtract 32, then divide by 1.8. So 95 °F is 35 °C.
d. $C = 1.8C + 32$, $-0.8C = 32$, $C =$ **−40**, so the point is $(-40, -40)$. Where the lines cross, F and C are equal. So -40 °C is the same as -40 °F.

Answer to Question 13

a. Hall A: $y = 40 + 5h$. Hall B: $y = 15h$.
b. $40 + 5h = 15h$, $10h = 40$, $h = 4$, cost = **£60** at $(4, 60)$.
c. 3 hours: A = $£55$, B = $£45$, so **B**. 6 hours: A = $£70$, B = $£90$, so **A**.
d. For a 4-hour hire both halls cost £60. Below 4 hours Hall B is cheaper and above 4 hours Hall A is cheaper.

Answer to Question 14

a. Yes: 0 litres costs £0. Buying no petrol costs nothing, so the point $(0, 0)$ is on the line.
b. $C = 1.5L$ The cost is the price per litre times the number of litres.
c. **1.5** (pounds per litre). The gradient is the price per litre, because each extra litre adds £1.50.
d. $63 \div 1.5 =$ **42 litres**. Divide the cost by the price per litre. Mia bought 42 litres.

Answer to Question 15

a. $2 \times m = -1$, so $m = -\frac{1}{2}$. The gradient of a perpendicular line is the negative reciprocal: turn 2 upside down and change the sign.
b. $y = -\frac{1}{2}x + c$: $1 = -2 + c$, $c = 3$. $y = -\frac{1}{2}x + 3$ Substitute the point $(4, 1)$ to find c. Check: $-\frac{1}{2} \times 4 + 3 = 1$.
c. $2x + 3 = -\frac{1}{2}x + 3$, $2.5x = 0$, $x = 0$, $y = 3$. $(0, 3)$ At the crossing point both equations give the same y.
d. The second line is $y = -\frac{1}{3}x + 2$, gradient $-\frac{1}{3}$. $3 \times \left(-\frac{1}{3}\right) = -1$ ✓. Rearrange the second equation to make y the subject. The product of the gradients is -1, so the lines are perpendicular.

Answer to Question 16

a. Between $x = 2$ and 4: $\frac{8}{2} = 4$. Between 4 and 6: $\frac{8}{2} = 4$. The gradients are equal, so it is a straight line.
b. $y = 4x + c$: $5 = 8 + c$, $c = -3$. $y = 4x - 3$ Substitute the point $(2, 5)$ to find c. Check: $4 \times 6 - 3 = 21$.
c. $4 \times 10 - 3 =$ **37** Substitute $x = 10$ into $y = 4x - 3$.
d. $-3 = 4x - 3$, so $x =$ **0**. Add 3 to both sides, so $4x = 0$.

Answer to Question 17

a. AB: $(4, 2)$. BC: $(5.5, 5)$. AC: $(2.5, 5)$. Each midpoint is the mean of the x values and the mean of the y values of the two ends.
b. $AC = \sqrt{3^2 + 6^2} = \sqrt{45} = 6.7$. $BC = \sqrt{3^2 + 6^2} = \sqrt{45} = 6.7$. So the triangle is **isosceles** (AC = BC).
c. Base $AB = 6$, height $= 8 - 2 = 6$. Area $= \frac{1}{2} \times 6 \times 6 =$ **18 square units**. AB is horizontal, so the perpendicular height is the difference in y between C and the line AB.
d. $m = \frac{8 - 2}{4 - 1} = 2$, $2 = 2 + c$, $c = 0$. $y = 2x$ Check with C: $2 \times 4 = 8$.

Answer to Question 18

a. PQ: $\frac{1}{4}$. SR: $\frac{5 - 4}{6 - 2} = \frac{1}{4}$. Use $m = \frac{\text{change in } y}{\text{change in } x}$ for each side.
b. QR: $\frac{5 - 1}{6 - 4} = 2$. PS: $\frac{4 - 0}{2 - 0} = 2$.
c. Both pairs of opposite sides have equal gradients, so they are parallel: **PQRS is a parallelogram**.
d. Midpoint of PR $= (3, 2.5)$. Midpoint of QS $= \left(\frac{4 + 2}{2}, \frac{1 + 4}{2}\right) = (3, 2.5)$ ✓.

Answer to Question 19

a. $x = 6 - x$, $x = 3$, $y = 3$. $(3, 3)$ Where the lines cross, the two y values are equal.
b. $y = x$ meets the y-axis at $(0, 0)$. $y = 6 - x$ meets it at $(0, 6)$.
c. Base on the y-axis $= 6$, height $= 3$. Area $= \frac{1}{2} \times 6 \times 3 =$ **9 square units**.
d. Sides: 6, $\sqrt{18}$, $\sqrt{18}$. Perimeter $= 6 + 2 \times 4.24 =$ **14.5**. Each sloping side has changes of 3 and 3, so its length is $\sqrt{3^2 + 3^2} = \sqrt{18}$.

Answer to Question 20

a. First: $y = -\frac{k}{2}x + 4$, gradient $-\frac{k}{2}$. Second: $y = 3x - 1$, gradient 3.

b. $-\frac{k}{2} = 3$, so $k = -6$. Parallel lines have equal gradients.

c. $\left(-\frac{k}{2}\right) \times 3 = -1$, $k = \frac{2}{3}$. So $k = \frac{2}{3}$. For perpendicular lines the product of the gradients is -1.

d. $2x + 2y = 8$ and $y = 3x - 1$: $2x + 6x - 2 = 8$, $8x = 10$, $x = 1.25$, $y = 2.75$. $(1.25, 2.75)$