Worksheet - Simultaneous equations

Year: Year 10 · 20

Show your working.

Question 1

Question 1 – Elimination by adding or subtracting *(Section A · Skills practice)*

Solve each pair of simultaneous equations by elimination. Check each answer in the other equation.

a. $x + y = 10$ and $x - y = 4$
b. $3x + 2y = 16$ and $x - 2y = 0$
c. $5x + y = 17$ and $2x + y = 8$
d. $4x + 3y = 13$ and $4x - y = 5$

Question 2

Question 2 – Elimination after multiplying one equation *(Section A · Skills practice)*

Multiply one equation so that the coefficients of one letter match, then eliminate. Solve.

a. $2x + 3y = 13$ and $x + y = 5$
b. $4x + 3y = 18$ and $2x + 5y = 16$
c. $3x - 2y = 7$ and $x + 4y = 7$
d. $5x + 2y = 24$ and $x - y = 2$

Question 3

Question 3 – Elimination after multiplying both equations *(Section A · Skills practice)*

Multiply both equations to make a pair of coefficients match. Solve.

a. $3x + 2y = 12$ and $5x - 3y = 1$
b. $2x + 5y = 16$ and $3x - 2y = 5$
c. $4x + 3y = 1$ and $3x + 2y = 0$
d. $7x - 2y = 17$ and $3x + 5y = 19$

Question 4

Question 4 – Substitution *(Section A · Skills practice)*

Solve each pair by substitution. Use brackets when you replace a letter.

a. $y = 2x + 1$ and $3x + y = 11$
b. $x = 3y - 2$ and $2x + y = 10$
c. $y = x - 4$ and $2x + 3y = 3$
d. $y = 5 - x$ and $y = 2x - 1$

Question 5

Question 5 – Solving graphically *(Section A · Skills practice)*

Use graphs to solve simultaneous equations. In each case the solution is where the two lines cross.

a. The lines $y = x + 1$ and $x + y = 7$ are drawn. State their point of intersection.
b. The lines $y = 2x - 1$ and $y = 5 - x$ cross at which point?
c. Explain why $y = 2x + 3$ and $y = 2x - 1$ have no solution.
d. Show that $(3, 4)$ satisfies both equations in part (a).

Question 6

Question 6 – Negative and fractional solutions *(Section A · Skills practice)*

Solve these pairs. Some answers are negative or fractions.

a. $2x - y = 7$ and $x + 3y = 0$
b. $x + y = 1$ and $x - y = 2$
c. $2x + y = 1$ and $x - y = 1$
d. $3x + 2y = 4$ and $6x + 4y = 8$. What do you notice?

Question 7

Question 7 – Cinema tickets *(Section B · Real-life practice)*

At a cinema, 2 adult tickets and 3 child tickets cost £28. 1 adult ticket and 4 child tickets cost £24.

a. Let a be the price of an adult ticket and c the price of a child ticket. Write two equations.
b. Solve them to find a and c.
c. Find the cost of a family ticket: 2 adults and 2 children.
d. A group buys 5 adult and 5 child tickets. How much do they pay?

Question 8

Question 8 – Coffee shop *(Section B · Real-life practice)*

At a café, 3 coffees and 2 teas cost £11.10. 2 coffees and 4 teas cost £11.80.

a. Let c pounds be the price of a coffee and t pounds the price of a tea. Write two equations.

b. Solve to find the price of a coffee and a tea.
c. Find the cost of 4 coffees and 1 tea.
d. Priya has £20. What is the greatest number of coffees she can buy with one tea?

Question 9

Question 9 – The trail mix *(Section B · Real-life practice)*

A shop mixes almonds and raisins. 3 kg of almonds and 2 kg of raisins cost £29. The total mass of 1 kg of almonds and 1 kg of raisins mixed together costs £11.

a. Let a and r be the prices per kilogram. Write two equations.
b. Solve them.
c. Find the cost of a mix of 2 kg almonds and 3 kg raisins.
d. What is the cost per kg of that mix?

Question 10

Question 10 – The rectangular garden *(Section B · Real-life practice)*

The perimeter of a rectangular lawn is 34 m. The length is 5 m more than the width.

a. Let l and w be the length and width. Write two equations.
b. Solve to find l and w.
c. Find the area of the lawn.
d. Turf costs £4.50 per m². Find the cost of turfing the lawn.

Question 11

Question 11 – Counting the coins *(Section B · Real-life practice)*

A collection box holds 30 coins. They are all 20p or 50p coins. The total value is £10.20.

a. Let a be the number of 20p coins and b the number of 50p coins. Write an equation for the number of coins and one for the value in pence.
b. Solve to find a and b.
c. Check that the value is right.
d. How many coins would be needed to make £10.20 using only 50p and 20p coins if there were 28 coins?

Question 12

Question 12 – Ages *(Section B · Real-life practice)*

A father is 3 times as old as his son. In 12 years' time the father will be twice as old as his son.

a. Let the son be s years and the father f years. Write two equations.

b. Solve them to find their ages now.
c. How old will each be in 12 years?
d. How old was the father when his son was born?

Question 13

Question 13 – Car hire *(Section B · Real-life practice)*

Company A charges a fixed £30 plus £0.20 per mile. Company B charges a fixed £20 plus £0.30 per mile. Let m be the miles and C the cost in pounds.

Miles	0	100	200
A: $C = 30 + 0.2m$	30	50	70
B: $C = 20 + 0.3m$	20	50	80

a. Write the two equations.
b. Solve them simultaneously to find the distance where the cost is equal.
c. State the cost at that distance.
d. Which company is cheaper for a 250-mile trip, and by how much?

Question 14

Question 14 – School trip transport *(Section B · Real-life practice)*

A school hires coaches and minibuses. 3 coaches and 2 minibuses seat 190 pupils. 2 coaches and 3 minibuses seat 160 pupils.

a. Write two equations in c (seats per coach) and m (seats per minibus).
b. Solve to find c and m.
c. How many pupils could go in 4 coaches and 1 minibus?
d. There are 230 pupils. Find one way to carry them using whole numbers of coaches and minibuses.

Question 15

Question 15 – Printing T-shirts *(Section C · Problem solving)*

Firm A charges a setup fee of £50 plus £4 per T-shirt. Firm B charges £20 plus £6 per T-shirt.

a. Write the cost equations for n T-shirts.
b. Find n for equal cost and the cost.
c. Draw (describe) the graphs and say which firm is cheaper for 10 shirts and for 25 shirts.
d. A club needs 40 shirts. How much does it save by choosing the cheaper firm?

Question 16

Question 16 – Number puzzles *(Section C · Problem solving)*

Form simultaneous equations and solve.

a. Two numbers add up to 47 and differ by 13. Find them.
b. A two-digit number has digits adding to 11. When the digits are reversed the number increases by 27. Find the number (tens digit t, units digit u).
c. The sum of two numbers is 20 and twice the first plus three times the second is 48. Find them.
d. Check part (b).

Question 17

Question 17 – Higher: a line and a circle *(Section C · Problem solving)*

Higher tier. Solve $x^2 + y^2 = 25$ and $y = x - 1$ by substitution.

a. Substitute to show that $x^2 - x - 12 = 0$.
b. Factorise and solve for x.
c. Find the two pairs of (x, y).
d. Explain what the solutions mean for the line and the circle.

Question 18

Question 18 – Higher: a line and a parabola *(Section C · Problem solving)*

Higher tier. A parabola has equation $y = x^2 - 4$ and a line has equation $y = 3x$.

a. Show that the intersections satisfy $x^2 - 3x - 4 = 0$.
b. Solve this equation.
c. Find the coordinates of the two points of intersection.
d. Is the line $y = -5$ meeting the parabola? Explain.

Question 19

Question 19 – Higher: finding the equation of a line *(Section C · Problem solving)*

Higher tier. A line has equation $ax + by = 7$ and passes through $(1, 2)$ and $(3, -1)$.

a. Write two equations in a and b.
b. Solve them.
c. Write down the equation of the line.
d. Find where the line crosses the y-axis.

Question 20

Question 20 – Challenge: how many solutions? *(Section D · Challenge)*

Consider the pair $2x + 3y = 5$ and $4x + 6y = k$.

a. For what value of k are the two equations the same line?
b. For what values of k are the lines parallel with no solution?
c. Describe the graphs in each case.
d. Solve the pair $2x + 3y = 5$ and $4x + 9y = k$ for x and y in terms of k.