Answers - Simultaneous equations

Year: Year 10

Answer to Question 1

a. Add: $2x = 14$, $x = 7$, $y = 3$. $x = 7$, $y = 3$ The y terms have opposite signs, so adding the equations eliminates y. Check: $7 - 3 = 4$.
b. Add: $4x = 16$, $x = 4$, then $4 - 2y = 0$, $y = 2$. $x = 4$, $y = 2$ The $-2y$ and $+2y$ cancel when we add. Check in the first equation: $12 + 4 = 16$.
c. Subtract: $3x = 9$, $x = 3$, then $15 + y = 17$, $y = 2$. $x = 3$, $y = 2$ Both equations contain $+y$, so subtracting eliminates y. Check: $6 + 2 = 8$.
d. Subtract: $4y = 8$, $y = 2$. Then $4x - 2 = 5$, $4x = 7$, $x = 1.75$. $x = 1.75$, $y = 2$ The $4x$ terms are the same, so subtracting the second equation from the first removes x. Check: $7 + 6 = 13$.

Answer to Question 2

a. Double the second: $2x + 2y = 10$. Subtract: $y = 3$, $x = 2$. $x = 2$, $y = 3$
b. Double the second: $4x + 10y = 32$. Subtract: $7y = 14$, $y = 2$, $4x = 12$, $x = 3$. $x = 3$, $y = 2$
c. Double the first: $6x - 4y = 14$. Add $x + 4y = 7$: $7x = 21$, $x = 3$, then $y = 1$. $x = 3$, $y = 1$
d. Double the second: $2x - 2y = 4$. Add to the first: $7x = 28$, $x = 4$, $y = 2$. $x = 4$, $y = 2$

Answer to Question 3

a. Multiply by 3 and 2: $9x + 6y = 36$ and $10x - 6y = 2$. Add: $19x = 38$, $x = 2$, $y = 3$. $x = 2$, $y = 3$
b. Multiply by 2 and 5: $4x + 10y = 32$ and $15x - 10y = 25$. Add: $19x = 57$, $x = 3$, $y = 2$. $x = 3$, $y = 2$
c. Multiply by 3 and 4: $12x + 9y = 3$ and $12x + 8y = 0$. Subtract: $y = 3$, then $4x + 9 = 1$, $x = -2$. $x = -2$, $y = 3$
d. Multiply by 5 and 2: $35x - 10y = 85$ and $6x + 10y = 38$. Add: $41x = 123$, $x = 3$, then $21 - 2y = 17$, $y = 2$. $x = 3$, $y = 2$

Answer to Question 4

a. $3x + (2x + 1) = 11$, $5x = 10$, $x = 2$, $y = 5$. $x = 2$, $y = 5$ Replace y by $2x + 1$ in the second equation, so only x is left. Check: $6 + 5 = 11$.
b. $2(3y - 2) + y = 10$, $7y = 14$, $y = 2$, $x = 4$. $x = 4$, $y = 2$ Replace x by $3y - 2$ and

expand the bracket: $6y - 4 + y = 10$. Check: $8 + 2 = 10$.

c. $2x + 3(x - 4) = 3$, $5x = 15$, $x = 3$, $y = -1$. $x = 3$, $y = -1$ Expand $3(x - 4) = 3x - 12$, then add 12 to both sides. Check: $6 - 3 = 3$.

d. $5 - x = 2x - 1$, $6 = 3x$, $x = 2$, $y = 3$. $x = 2$, $y = 3$ Both expressions equal y, so they are equal to each other. Check: $2 \times 2 - 1 = 3$.

Answer to Question 5

a. Substitute: $x + x + 1 = 7$, $x = 3$, $y = 4$. $(3, 4)$ The point where two lines cross lies on both lines, so it is the solution of both equations.

b. $2x - 1 = 5 - x$, $3x = 6$, $x = 2$, $y = 3$. $(2, 3)$ At the crossing point the two y values are equal. Check: $5 - 2 = 3$.

c. Both lines have gradient 2 but different intercepts (3 and -1), so they are **parallel** and never meet. Parallel lines never cross, so the simultaneous equations have no solution.

d. $4 = 3 + 1$ ✓ and $3 + 4 = 7$ ✓. The point fits both equations, so $(3, 4)$ is the point of intersection.

Answer to Question 6

a. From the second, $x = -3y$. Then $-6y - y = 7$, $y = -1$, $x = 3$. $x = 3$, $y = -1$

b. Add: $2x = 3$, $x = 1.5$, $y = -0.5$. $x = 1.5$, $y = -0.5$ Check: $1.5 + (-0.5) = 1$ and $1.5 - (-0.5) = 2$.

c. Add: $3x = 2$, $x = \frac{2}{3}$, $y = -\frac{1}{3}$. $x = \frac{2}{3}$, $y = -\frac{1}{3}$ Check: $\frac{4}{3} - \frac{1}{3} = 1$ and $\frac{2}{3} + \frac{1}{3} = 1$.

d. The second equation is twice the first, so they are the **same line**: there are infinitely many solutions.

Answer to Question 7

a. $2a + 3c = 28$ and $a + 4c = 24$. Each equation turns one sentence into algebra: the number of tickets times the price, added together.

b. Double the second: $2a + 8c = 48$. Subtract: $5c = 20$, $c = 4$, then $a = 8$. **Adult £8, child £4**. Doubling the second equation makes the a terms match, so subtracting removes a. Check: $16 + 12 = 28$ and $8 + 16 = 24$.

c. $2 \times 8 + 2 \times 4 =$ **£24**. Substitute the two prices. A family ticket costs £24.

d. $5 \times 8 + 5 \times 4 =$ **£60**. Five adults and five children cost $5 \times (8 + 4) = 5 \times 12$. The group pays £60.

Answer to Question 8

a. $3c + 2t = 11.10$ and $2c + 4t = 11.80$. Write one equation for each order: the number of drinks times the price, added together.

b. Double the first: $6c + 4t = 22.20$. Subtract the second: $4c = 10.40$, $c = 2.60$. Then $2t = 11.10 - 7.80 = 3.30$, $t = 1.65$. **Coffee £2.60, tea £1.65**. Doubling the first equation makes the t terms match. Check: $5.20 + 6.60 = 11.80$.
c. $4 \times 2.60 + 1.65 = 10.40 + 1.65 =$ **£12.05**. Substitute the two prices. Four coffees and a tea cost £12.05.
d. $20 - 1.65 = 18.35$, $18.35 \div 2.60 = 7.06$, so **7 coffees**. Pay for the tea first, then see how many whole coffees fit in the money left. We round down because she cannot buy part of a coffee.

Answer to Question 9

a. $3a + 2r = 29$ and $a + r = 11$. One equation uses the 3 kg and 2 kg mix, the other uses 1 kg of each.
b. From the second, $r = 11 - a$: $3a + 22 - 2a = 29$, so $a = 7$ and $r = 4$. **Almonds £7/kg, raisins £4/kg.** Substitute $r = 11 - a$ into the first equation. Check: $21 + 8 = 29$.
c. $2 \times 7 + 3 \times 4 = 14 + 12 =$ **£26**. Multiply each mass by its price per kilogram and add.
d. $26 \div 5 =$ **£5.20 per kg**. The mix has a mass of $2 + 3 = 5$ kg, so divide the cost by 5.

Answer to Question 10

a. $2l + 2w = 34$ (so $l + w = 17$) and $l = w + 5$. The perimeter is two lengths plus two widths, and the second sentence gives the link between l and w.
b. $w + 5 + w = 17$, $2w = 12$, $w = 6$, $l = 11$. **Length 11 m, width 6 m.** Substitute $l = w + 5$ into $l + w = 17$. Check: $2 \times 11 + 2 \times 6 = 34$.
c. $11 \times 6 =$ **66 m²**. The area of a rectangle is length times width.
d. $66 \times 4.50 =$ **£297**. Multiply the area by the price per square metre. The turf costs £297.

Answer to Question 11

a. $a + b = 30$ and $20a + 50b = 1020$.
b. $b = 30 - a$: $20a + 1500 - 50a = 1020$, $-30a = -480$, $a = 16$, $b = 14$. **16 twenty-pence and 14 fifty-pence coins.**
c. $16 \times 20 = 320$ and $14 \times 50 = 700$, total 1020p = **£10.20** ✓
d. $20a + 50(28 - a) = 1020$ gives $-30a = -380$, $a = 12.67$, not a whole number, so **it is impossible**.

Answer to Question 12

a. $f = 3s$ and $f + 12 = 2(s + 12)$. The father's age now is three times his son's. In 12 years both ages increase by 12.
b. $3s + 12 = 2s + 24$, $s = 12$, $f = 36$. **Son 12, father 36.** Substitute $f = 3s$ into the second equation, then solve for s.
c. Son **24**, father **48** ($48 = 2 \times 24$ ✓). Add 12 to each age now. The father will be twice as old, as the question says.
d. $36 - 12 =$ **24 years old**. The age difference never changes, so the father was 24 when his son was born.

Answer to Question 13

a. $C = 30 + 0.2m$ and $C = 20 + 0.3m$. Each company charges a fixed amount plus a cost for each mile.
b. $30 + 0.2m = 20 + 0.3m$, $10 = 0.1m$, $m = 100$ **miles**. The costs are equal when the two expressions for C are equal. Subtract $0.2m$ and 20 from both sides.
c. $30 + 0.2 \times 100 =$ **£50**. Substitute $m = 100$ into either equation. Both companies charge £50.
d. 250 miles: A $= 30 + 50 = £80$, B $= 20 + 75 = £95$. **A is cheaper by £15**. After 100 miles company A becomes cheaper because its cost per mile is lower.

Answer to Question 14

a. $3c + 2m = 190$ and $2c + 3m = 160$.
b. Multiply by 3 and 2: $9c + 6m = 570$ and $4c + 6m = 320$. Subtract: $5c = 250$, $c = 50$, then $m = 20$. **Coach 50, minibus 20.**
c. $4 \times 50 + 20 =$ **220 pupils**. Four coaches seat 200 pupils and one minibus seats 20.
d. For example 4 coaches and 2 minibuses: $200 + 40 = 240 \geq 230$. Or 3 coaches and 4 minibuses: $150 + 80 = 230$ exactly.

Answer to Question 15

a. $C_A = 50 + 4n$ and $C_B = 20 + 6n$.
b. $50 + 4n = 20 + 6n$, $30 = 2n$, $n = 15$, cost $= 50 + 60 =$ **£110**.
c. 10 shirts: A $= £90$, B $= £80$, so **B is cheaper**. 25 shirts: A $= £150$, B $= £170$, so **A is cheaper**. The lines cross at (15, 110).
d. 40 shirts: A $= 210$, B $= 260$. A saves **£50**.

Answer to Question 16

a. $x + y = 47$ and $x - y = 13$. Add: $2x = 60$, $x = 30$, $y = 17$. **30 and 17**.
b. $t + u = 11$ and $(10u + t) - (10t + u) = 27$, so $9u - 9t = 27$ and $u - t = 3$. Add to

$t + u = 11$: $2u = 14$, $u = 7$, $t = 4$. The number is **47**.

c. $x + y = 20$ and $2x + 3y = 48$. $y = 20 - x$: $2x + 60 - 3x = 48$, $x = 12$, $y = 8$. **12 and 8**.

d. Reversed number 74. $74 - 47 = 27$ ✓. Digit sum $4 + 7 = 11$ ✓.

Answer to Question 17

a. $x^2 + (x - 1)^2 = 25$, $2x^2 - 2x + 1 = 25$, $2x^2 - 2x - 24 = 0$, so $x^2 - x - 12 = 0$.

b. $(x - 4)(x + 3) = 0$, so $x = 4$ or $x = -3$.

c. $x = 4$: $y = 3$. $x = -3$: $y = -4$. $(4, 3)$ **and** $(-3, -4)$

d. The line cuts the circle (radius 5, centre the origin) at two points; both pairs satisfy $x^2 + y^2 = 25$ ($16 + 9$ and $9 + 16$).

Answer to Question 18

a. $x^2 - 4 = 3x$, so $x^2 - 3x - 4 = 0$.

b. $(x - 4)(x + 1) = 0$, so $x = 4$ or $x = -1$.

c. $x = 4$: $y = 12$. $x = -1$: $y = -3$. $(4, 12)$ **and** $(-1, -3)$

d. $x^2 - 4 = -5$ gives $x^2 = -1$, which has no real solution, so **no**: the lowest point of the parabola is $y = -4$.

Answer to Question 19

a. $a + 2b = 7$ and $3a - b = 7$. The line passes through each point, so substitute $x = 1$, $y = 2$ and then $x = 3$, $y = -1$ into $ax + by = 7$.

b. $b = 3a - 7$: $a + 6a - 14 = 7$, $7a = 21$, $a = 3$, $b = 2$. Make b the subject of the second equation and substitute it into the first. Check: $3 + 4 = 7$ and $9 - 2 = 7$.

c. $3x + 2y = 7$ Substitute $a = 3$ and $b = 2$ into $ax + by = 7$.

d. $x = 0$: $2y = 7$, so $(0, 3.5)$. Every point on the y-axis has $x = 0$. The line crosses the y-axis at $(0, 3.5)$.

Answer to Question 20

a. The second equals twice the first when $k =$ **10**.

b. They have the same gradient, so for any other k ($k \neq 10$) the lines are parallel and never meet.

c. $k = 10$: one line (infinitely many solutions). $k \neq 10$: two parallel lines (no solutions).

d. Double the first: $4x + 6y = 10$. Subtract from the second: $3y = k - 10$, $y = \frac{k - 10}{3}$. Then $2x = 5 - (k - 10) = 15 - k$, $x = \frac{15 - k}{2}$.