Test answers - Sequences

Year: Year 10 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. Difference 4 (1), $\mathbf{4n + 5}$ (1) The common difference is 4, and the zero term is $9 - 4 = 5$. Check: $4 \times 2 + 5 = 13$.
b. $4n + 5 = 150$, $4n = 145$, $n = 36.25$ (1), not a whole number so **150 is not a term** (1) Set the nth term equal to 150 and solve. A term number must be a whole number, so 150 is not a term of the sequence.

The 36th term is 149 and the 37th is 153.

Answer to Question 2 (4 marks)

a. Difference 2 (1), $2n + 2$ (1) Each extra table adds 2 seats, and the zero term is $4 - 2 = 2$. Check: $2 \times 3 + 2 = 8$.
b. $2n + 2 \geq 50$, $n \geq 24$ (1), so **24 tables** (1) Solve the inequality: $2n \geq 48$, so $n \geq 24$. Exactly 24 tables seat 50 people.

In (a), check one more pattern: four tables give $2 \times 4 + 2 = 10$ seats, since each end seats one person and each long side seats one per table. In (b), the answer must be a whole number of tables, and 23 tables would seat only 48 people.

Answer to Question 3 (4 marks)

a. Ratio **3** (1); next terms **135, 405** (1) Divide any term by the one before: $15 \div 5 = 3$. Then $45 \times 3 = 135$ and $135 \times 3 = 405$.
b. 5×3^7 (1) $= 5 \times 2187 =$ **10 935** (1) The nth term is $5 \times 3^{n-1}$, so the 8th term is 5×3^7. The 8th term is 10 935.

A common error in (b) is to work out 5×3^8, using the term number as the power; the first term already uses 3^0. Another error is 15^7. Check: listing gives 5, 15, 45, 135, 405, 1215, 3645, 10 935.

Answer to Question 4 (4 marks)

a. Difference 6 (1), **£**$(6n + 14)$ (1) The common difference is 6, and the zero term is $20 - 6 = 14$. Check: $6 \times 3 + 14 = 32$.
b. Week 10: 74. Sum $= \frac{10}{2}(20 + 74)$ (1) $=$ **£470** (1) In week 10 she saves $6 \times 10 + 14 = 74$ pounds. Use the sum formula with first term 20 and last term 74. She has saved £470.

Check (b) by adding: $20 + 26 + 32 + \cdots + 74$ has ten terms with an average of 47, and $47 \times 10 = 470$.

Answer to Question 5 (4 marks)

a. First differences 7, 11, 15; second differences 4, 4 (constant) (1)
b. $2n^2$ gives 2, 8, 18, 32; remainders 2, 3, 4, 5 so $n + 1$ (1). $2n^2 + n + 1$ (1) Half of the second difference 4 gives $2n^2$.
c. $2 \times 225 + 15 + 1 =$ **466** (1) Substitute $n = 15$: $15^2 = 225$. The 15th term is 466.

In (a), a constant second difference proves the sequence is quadratic. Check (b) with $n = 4$: $2 \times 16 + 4 + 1 = 37$, which matches the fourth term given in the question.