Worksheet - Sequences

Year: Year 10 · 20

Show your working.

Question 1

Question 1 – Continuing sequences *(Section A · Skills practice)*

Write down the next two terms of each sequence and describe the rule in words.

a. 5, 9, 13, 17, ...
b. 80, 40, 20, 10, ...
c. 2, 6, 18, 54, ...
d. 1, 4, 9, 16, ...

Question 2

Question 2 – The nth term of an arithmetic sequence *(Section A · Skills practice)*

Find the nth term of each arithmetic sequence.

a. 7, 11, 15, 19, ...
b. 20, 17, 14, 11, ...
c. -3, 2, 7, 12, ...
d. Use the nth term from part (a) to find the 50th term.

Question 3

Question 3 – Finding terms from the nth term *(Section A · Skills practice)*

Write down the first four terms of each sequence.

a. $5n - 2$
b. $n^2 + 3$
c. 2^n
d. $n(n + 1)$

Question 4

Question 4 – Is it in the sequence? *(Section A · Skills practice)*

Decide whether each number is a term of the sequence. If it is, state which term.

a. Is 100 in the sequence $3n + 1$?
b. Is 150 in the sequence $4n - 2$?
c. Is 75 in the sequence $6n + 1$?
d. Is 203 in the sequence $4n + 3$?

Question 5

Question 5 – Geometric sequences *(Section A · Skills practice)*

Each sequence is geometric (multiplying by a fixed number each time).

a. 3, 6, 12, 24, ... Find the common ratio and the 10th term.
b. 160, 80, 40, ... Find the common ratio and the 6th term.
c. 1, 3, 9, 27, ... Find the first term bigger than 1000.
d. 2, -4, 8, ... Find the next two terms.

Question 6

Question 6 – Special sequences *(Section A · Skills practice)*

Name each sequence and write down the next two terms.

a. 1, 3, 6, 10, 15, ...
b. 1, 8, 27, 64, ...
c. 1, 1, 2, 3, 5, 8, ...
d. 2, 3, 5, 7, 11, ...

Question 7

Question 7 – Matchstick triangles *(Section B · Real-life practice)*

Triangles are made from matchsticks in a row. Pattern 1 has 1 triangle (3 matches), pattern 2 has 2 triangles (5 matches) and pattern 3 has 3 triangles (7 matches).

Pattern	1	2	3	4
Matches	3	5	7	?

a. How many matches are in pattern 4 and pattern 5?
b. Find the nth term for the number of matches.
c. How many matches are needed for pattern 25?
d. Adam has 100 matches. What is the biggest pattern he can make, and how many matches are left?

Question 8

Question 8 – Saving up *(Section B · Real-life practice)*

Ellie has £50 in a savings jar and adds £15 each week.

a. Write down her savings after weeks 1, 2 and 3.
b. Find a formula for her savings after n weeks.
c. How much does she have after 20 weeks?
d. After how many weeks does she first have more than £500?

Question 9

Question 9 – Patio border *(Section B · Real-life practice)*

A square patio is made from $n \times n$ white paving slabs. A border of grey slabs goes round the outside. For $n = 1$ the border has 8 grey slabs, for $n = 2$ it has 12, for $n = 3$ it has 16.

a. Find the number of grey slabs for $n = 4$.
b. Show that the nth term is $4n + 4$.
c. A patio has 60 grey border slabs. Find n and the number of white slabs.
d. Explain how $4n + 4$ can be seen from the picture: four sides of n slabs and four corners.

Question 10

Question 10 – Tables at the party *(Section B · Real-life practice)*

At a party, one table seats 6 people. Each extra table joined at the end adds 4 more seats.

a. How many people can sit at 2 tables and 3 tables?
b. Find the nth term for the number of seats.
c. How many tables are needed for 90 people?
d. There are 12 tables joined end to end. How many seats is that?

Question 11

Question 11 – Bacteria doubling *(Section B · Real-life practice)*

A scientist starts with 100 bacteria. The number doubles every hour. Calculator allowed.

Hours	0	1	2	3
Bacteria	100	200	400	800

a. Write a formula for the number of bacteria after n hours.
b. Find the number after 8 hours.
c. After how many hours is the number first over 1 million?
d. Is this an arithmetic or geometric sequence? Explain.

Question 12

Question 12 – Growing savings *(Section B · Real-life practice)*

Sam invests £1000 at 5% compound interest per year. The amount at the end of each year forms a sequence.

a. Write down the amounts at the end of years 1, 2 and 3.

b. What is the common ratio?
c. Find the amount after 5 years to the nearest penny.
d. After how many years does the amount first exceed £1500?

Question 13

Question 13 – Stacking tins *(Section B · Real-life practice)*

A shop stacks tins in a triangle. The top row has 1 tin, the next 2, then 3, and so on.

a. How many tins are in the 6th row?
b. How many tins are in a stack of 6 rows (triangular number)?
c. The formula for the total in n rows is $\frac{n(n+1)}{2}$. Use it for 10 rows.
d. The shop has 200 tins. How many full rows can it make, and how many tins are left over?

Question 14

Question 14 – Terraced seating *(Section B · Real-life practice)*

In a small theatre the front row has 12 seats and each row behind has 2 more seats than the row in front of it.

a. How many seats are in row 5?
b. Find the nth term.
c. There are 20 rows. How many seats are in the back row?
d. Find the total number of seats in the first 5 rows.

Question 15

Question 15 – Higher: quadratic sequences *(Section C · Problem solving)*

Higher tier. For a quadratic sequence the second differences are constant. If the second difference is $2a$, the nth term starts with an^2.

a. Find the nth term of 3, 9, 19, 33, ...
b. Find the nth term of 2, 7, 14, 23, ...
c. Find the 20th term of the sequence in part (a).
d. Show that the sequence in part (b) contains the number 62 or explain why not.

Question 16

Question 16 – Proof with sequences *(Section C · Problem solving)*

Use algebra to prove each statement about sequences.

a. Show that every term of the sequence $4n + 3$ is odd.

b. Show that the sum of the first n odd numbers $1 + 3 + \ldots + (2n - 1)$ is n^2 for $n = 1, 2, 3, 4$.
c. Show that the difference between the squares of any two consecutive terms of the sequence n is odd: $(n + 1)^2 - n^2$.
d. Show that every term of the sequence $6n - 1$ is one less than a multiple of 6 and never divisible by 3.

Question 17

Question 17 – Fibonacci-type sequences *(Section C · Problem solving)*

In a Fibonacci-type sequence each term after the second is the sum of the two terms before it. The first two terms are a and b.

a. Write the third, fourth and fifth terms in terms of a and b.
b. The 4th term is 17 and the 5th term is 27. Form two equations and solve them to find a and b.
c. Write down the first six terms.
d. Find the 8th term.

Question 18

Question 18 – Geometric sequences with unknowns *(Section C · Problem solving)*

A geometric sequence has second term 12 and fifth term 96.

a. Let the first term be a and the common ratio r. Write two equations.
b. Divide to find r^3 and then r.
c. Find the first term and the first five terms.
d. Find the 10th term.

Question 19

Question 19 – Adding up a sequence *(Section C · Problem solving)*

Calculator allowed. The sum of the first n terms of an arithmetic sequence is $S_n = \frac{n}{2}(\text{first} + \text{last})$.

a. Add the whole numbers from 1 to 100 by pairing: $1 + 100$, $2 + 99$, and so on.
b. The sequence $3n + 2$ starts 5, 8, 11, ... Find the 20th term.
c. Find the sum of the first 20 terms of $3n + 2$.
d. How many terms of $3n + 2$ are needed for the sum to exceed 1000?

Question 20

Question 20 – Challenge: the dot patterns *(Section D · Challenge)*

A pattern of dots has 5, 12, 23, 38 dots in the first four patterns.

a. Find the first and second differences.
b. Find the nth term.
c. Find the number of dots in pattern 10.
d. What is the first pattern with more than 1000 dots?